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Circuit Breaker Transient Recovery Voltage Analysis with Shunt Capacitor Bank Configurations

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by

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REPORT

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Dedicated to my Parents.

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Statement of Ethics and Academic Integrity

I certify that I have completed the online ethics training modules, particularly the Academic Integrity Module[1], of the University of Texas at Austin - Graduate School. I fully understand, and I am familiar with the University policies and regulations relating to Academic Integrity, and the Academic Policies and Procedures[2]. I also attest that this report is the result of my own original work and efforts. Any ideas of other authors, whether or not they have been published or otherwise disclosed, are fully acknowledged and properly referenced. I also acknowledge the thoughts, direction, and supervision of my research advisor, Prof. S. Santoso.

[1]The University of Texas at Austin - Graduate Schools online ethics training modules, http://www.utexas.edu/ogs/ethics/, and the ethics training on academic integrity, http://www.utexas.edu/ogs/ethics/transcripts/academic.html, accessed on Nov. 18, 2010.

[2]The University of Texas at Austin, General Information, 2006-2007, Chapter 11, Sec.11 101, http://www.utexas.edu/student/registrar/catalogs/gi06-07/app/appc11.html

Circuit Breaker Transient Recovery Voltage Analysis with Shunt Capacitor Bank Configurations

by

Anirudh Guha, MSE
The University of Texas at Austin, 2010
SUPERVISOR: Surya Santoso

Transient Recovery Voltage (TRV) is an important consideration in the selection and installation of circuit breakers with appropriate ratings. Capacitor banks with inrush current limiting reactors are an integral part of the power system. Capacitor banks with inrush reactors on the load side terminal of the capacitor breaker alter the TRV seen across the breaker and it is critical to carry out the TRV analysis to prevent circuit breaker failure. TRV analysis has been performed for various capacitor bank - inrush reactor configurations, with the fault occurring at different terminals on the load side. Analytical solutions have been presented for both single-phase and three-phase ungrounded capacitor banks. Neutral displacement voltage of three-phase ungrounded capacitor banks result in increased stress across the breaker. Results have been validated with PSCAD simulation and MATLAB plots.

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Chapter 1

Introduction

Shunt capacitor banks are used for reactive power compensation and are an integral part of the power system. But there are a few important issues that arise with shunt capacitor bank installations in the power system. The interaction between the system reactance and the shunt capacitor banks lead to transient voltages and currents during energization of the capacitor banks. The high magnitude and frequency of the inrush currents may exceed the circuit breaker ratings and this results in the breaker failure. Hence this necessitates for synchronous closing control, pre-insertion resistors, or the inclusion of inrush current limiting reactors in series with the capacitor banks in order to minimize the inrush current.

The inclusion of inrush reactors in the system can lead to a few issues with regard to de-energization of the capacitor bank and clearing of a fault that has occurred at the load side of the breaker. Transients initiated when the breaker opens can lead to breaker failure if the breaker ratings are exceeded. The magnitude and the rate of rise of the recovery voltage are important parameters to be analyzed. The presence of capacitor banks on the load side of the breaker also result in a possibility of restrike leading voltage build up across the capacitor bank and thus failed de-energization. Hence recovery voltage analysis of the circuit breaker become very important with the presence of capacitor banks and inrush reactors.

Three-phase capacitor banks are ungrounded to prevent the flow of the zero sequence current component. The neutral point is floating and there exists a neutral displacement voltage with respect to ground during an unbalance condition. This leads to an increased transient recovery voltage across the circuit breaker. Hence a breaker with a higher rating would be required.

An analysis of the recovery voltage across the breaker in single-phase and three-phase ungrounded systems with breaker load side capacitor banks and inrush reactors has been presented. The effect of the relative position of the inrush reactors with regard to the breaker load side terminal and capacitor bank has been analyzed in the forthcoming chapters.

1.1 Estimation of Transient Voltage across Stray Capacitance at the Source Side Breaker Terminal - Series LC Circuit

The transient voltage initiated across the stray capacitance to ground at the source side breaker terminals is estimated by considering the single phase circuit shown in Figure 1.1.

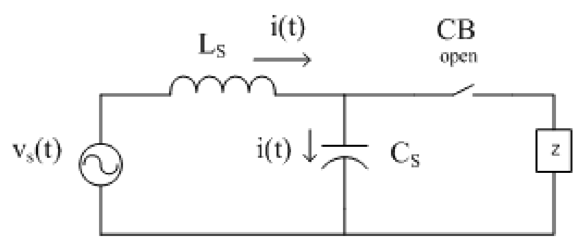

Figure 1.1: Basic Series LC Circuit for TRV Analysis

The generalized transient voltage across the capacitor in the circuit is derived first. This result is then used in calculation of the TRV during the de-energization of the capacitor bank in the absence of a fault and is followed by TRV calculations during the clearing of fault occurring at different locations on the load side in the subsequent chapters.

The circuit breaker de-energizes at time $t = 0$ and the source voltage is represented as $v_S(t) = V_m cos(\omega t)$. The voltage is maximum when the circuit breaker opens at the current zero. The transient occurs due to the change in the steady state voltage conditions across the stray capacitance. The differential equation governing the transient is given as

$$V_m cos(\omega t) = L_S \frac{di(t)}{dt} + v_{C_S}(t) \tag{1.1}$$

where

$$i(t) = C_S \frac{dv_{C_S}(t)}{dt} \tag{1.2}$$

Substituting Eq (1.2) in Eq (1.1), yields

$$V_m cos(\omega t) = L_S C_S \frac{d^2 v_{C_S}(t)}{dt^2} + v_{C_S}(t) \tag{1.3}$$

Let us define $\omega_0 = \frac{1}{\sqrt{L_S C_S}}$, and rewrite Eq (1.3).

$$\frac{d^2 v_{C_S}(t)}{dt^2} + {\omega_0}^2 v_{C_S}(t) = {\omega_0}^2 V_m cos(\omega t) \tag{1.4}$$

Taking the Laplace transform on both sides of Eq (1.4),

$$s^2 V_{C_S}(s) - s v_{C_S}(0) - {v_{C_S}}'(0) + {\omega_0}^2 V_{C_S}(s) = {\omega_0}^2 V_m \frac{s}{s^2 + \omega^2} \tag{1.5}$$

The circuit breaker opens at $t = 0$ and interrupts the current.

$$i(0) = C{v_{C_S}}'(0) = 0 \Rightarrow {v_{C_S}}'(0) = 0 \tag{1.6}$$

Using results given in Eq (1.6), Eq(1.5) becomes

$$\left(s^2 + {\omega_0}^2\right) V_{C_S}(s) = {\omega_0}^2 V_m \frac{s}{s^2 + \omega^2} + s v_{C_S}(0) \tag{1.7}$$

$$V_{C_S}(s) = {\omega_0}^2 V_m \frac{s}{\left(s^2 + \omega^2\right)\left(s^2 + {\omega_0}^2\right)} + v_{C_S}(0) \frac{s}{\left(s^2 + {\omega_0}^2\right)} \tag{1.8}$$

Taking the inverse Laplace transform on both sides of Eq (1.8),

$$v_{C_S}(t) = \frac{{\omega_0}^2 V_m}{\omega_0^2 - \omega^2} \left[cos\omega t - cos\omega_0 t\right] + v_{C_S}(0) cos\omega_0 t \tag{1.9}$$

$${\omega_0}^2 >> \omega^2 \Rightarrow \frac{{\omega_0}^2}{\omega_0^2 - \omega^2} \approx 1$$

$$v_{C_S}(t) = V_m \left[cos\omega t - cos\omega_0 t\right] + v_{C_S}(0) cos\omega_0 t \tag{1.10}$$

$$v_{C_S}(t) = V_m cos\omega t - \left[V_m - v_{C_S}(0)\right] cos\omega_0 t \quad \forall t \geq 0 \tag{1.11}$$

Equation (1.11) is the generalized expression for voltage developed across the stray capacitance at the source side terminal of the breaker after the opening of the circuit breaker. This result is used henceforth in subsequent sections where initial conditions pertaining to each case are applied to this equation to get transient voltages across the stray capacitance specific to the case.

1.2 Estimation of Transient Voltage across a L||Cp Element on the Load Side of the Breaker - Parallel LC Circuit

The transient voltage initiated across the L||Cp element on the load side of the breaker is estimated by considering the circuit shown in Figure 1.2. The circuit breaker opens at current zero, at time $t = 0$ and there is no current flowing into the load after the breaker opens. The charge stored in the capacitor leads to oscillations in the L||Cp element.

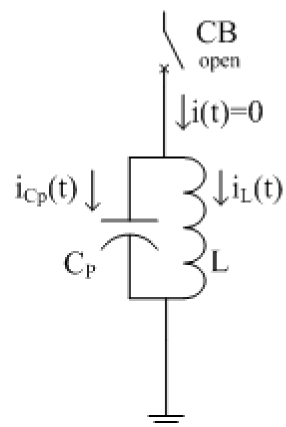

Figure 1.2: Basic Parallel LC Circuit for TRV Analysis

The current flowing through the circuit breaker is zero after the opening of the breaker at $t = 0$.

$$i_{C_P}(t) + i_L(t) = 0 \tag{1.12}$$

$$v_{C_P}(t) = L\frac{di_L}{dt} = -L\frac{di_{C_P}(t)}{dt} \tag{1.13}$$

$$i_{C_P}(t) = C_P \frac{dv_{C_P}}{dt} \tag{1.14}$$

Substituting Eq (1.14) in Eq (1.13), yields

$$v_{C_P}(t) = -LC_P \frac{d^2 v_{C_P}}{dt^2} \tag{1.15}$$

Let us define $\omega_P = \frac{1}{\sqrt{LC_P}}$, and rewrite Eq (1.15).

$$v_{C_P}(t) = -\frac{1}{{\omega_P}^2} \frac{d^2 v_{C_P}}{dt^2} \tag{1.16}$$

$$\frac{d^2 v_{C_P}}{dt^2} + {\omega_P}^2 v_{C_P}(t) = 0 \tag{1.17}$$

Taking the Laplace transform on both sides of Eq (1.17),

$$s^2 V_{C_P}(s) - s v_{C_P}(0) - {v_{C_P}}'(0) + {\omega_P}^2 V_{C_P}(s) = 0 \tag{1.18}$$

Using current division,

$$i_{C_P}(0) = i(0) \frac{X_L}{X_L - X_{C_P}} \tag{1.19}$$

But,

$$i(0) = 0 \Rightarrow i_{C_P}(0) = 0 \tag{1.20}$$

Therefore,

$${v_{C_P}}'(0) = \frac{i_{C_P}(0)}{C_P} = 0 \tag{1.21}$$

Using the result given in Eq (1.21), Eq (1.18) becomes

$$\left(s^2 + {\omega_P}^2\right) V_{C_P}(s) = s v_{C_P}(0) \tag{1.22}$$

$$V_{C_P}(s) = v_{C_P}(0) \frac{s}{\left(s^2 + {\omega_P}^2\right)} \tag{1.23}$$

Taking the inverse Laplace transform on both sides of Eq (1.23),

$$v_{C_P}(t) = v_{C_P}(0) cos \omega_P t \quad \forall t \geq 0 \tag{1.24}$$

The result given by Equation (1.24) is used in calculation of the TRV during de-energization and fault clearing with L||Cp element on the load side of the breaker.

Chapter 2

Single-phase Capacitor Banks

The transient recovery voltage analysis for single-phase capacitor bank configurations has been presented in this chapter. The recovery voltage analysis is first carried out for de-energization of the various capacitor bank - inrush reactor configurations. This is followed by the analysis of the TRV initiated when the circuit breaker opens to clear the fault. The fault is assumed to be a bolted fault to ground and hence the fault resistance is zero. Analysis has been presented for faults occurring at various possible locations of the different capacitor bank configurations. The derived results have been supported by PSCAD and MATLAB simulations.

2.1 De-energization of Capacitor Bank

The circuit breaker effectively interrupts the flow of current at the current zero to de-energize the capacitor bank. The load is predominantly capacitive and hence before the breaker opening, the line current approximately leads the source voltage by 90°. The source voltage is assumed to be $v_S(t) = V_m cos\omega t$. When the breaker opens at $t = 0$, voltage is at the positive maximum V_m while the current is at the zero crossing, where current goes from positive to negative.

The de-energization analysis of a single-phase capacitor bank with no inrush reactor and with inrush reactor is presented in the following sections.

2.1.1 Capacitor Bank without Inrush Reactor

The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at $t = 0$ is given by Eq (1.11).

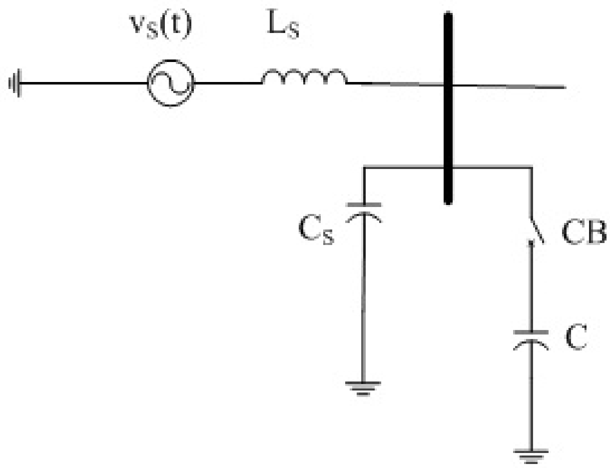

Figure 2.1: Capacitor De-energization without Inrush Reactor

$$v_{C_S}(t) = V_m cos\omega t - [V_m - v_{C_S}(0)]\, cos\omega_0 t \quad \forall t \geq 0 \tag{2.1}$$

Neglecting the voltage drop across the source reactance, the voltage at the bus is the peak voltage at current zero. The capacitor bank retains this voltage after the opening of the circuit breaker.

$$v_C(t) = v_C(0) = v_{C_S}(0) \approx V_m \quad \forall t \geq 0 \tag{2.2}$$

$$v_{TRV}(t) = v_{C_S}(t) - v_C(t) \tag{2.3}$$

$$v_{TRV}(t) = V_m cos\omega t - V_m \tag{2.4}$$

$$v_{TRV}(t) = V_m \left(cos\omega t - 1\right) \qquad \forall t \geq 0^+ \tag{2.5}$$

$$v_{TRV}(0^+) = 0 \tag{2.6}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{2.7}$$

$$v_{TRV}(0^-) = v_{TRV}(0^+) = v_{TRV}(0) = 0 \tag{2.8}$$

Validation

$$\begin{aligned} L_s &= 5 \ \text{mH} \\ C_s &= 0.29 \ \mu\text{F} \\ C &= 8.4239 \ \mu\text{F} \\ V_m &= 93.897 \ \text{kV} \\ \omega &= 376.99 \ \text{rad/sec} \end{aligned}$$

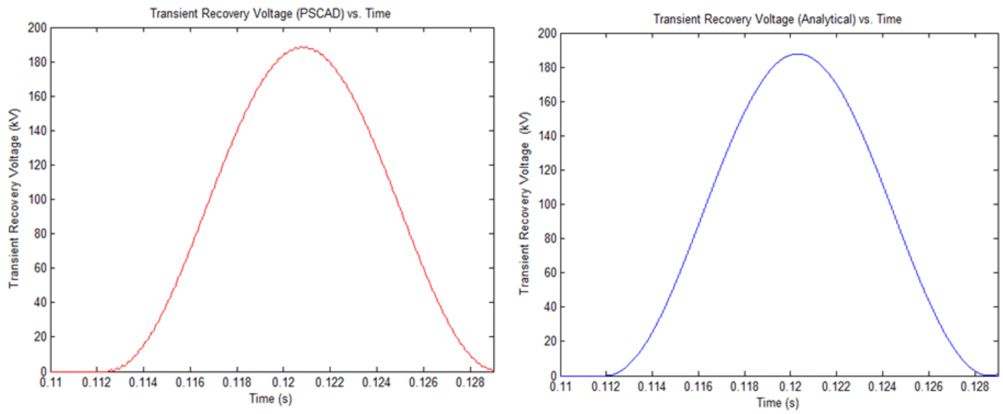

Figure 2.2: PSCAD and MATLAB Validation, Capacitor De-energization without Inrush Reactor

$$
\begin{aligned}
t_{open} &= 0.112 \text{ sec} \\
\forall t &\geq t_{open} \\
v_{TRV}(t) &= -V_m \left(cos\omega \left(t - t_{open} \right) - 1 \right) \\
v_{TRV}(t_{open}) &= 0 \\
v_{TRV,Peak} &= 2V_m = 187.794 \text{ kV}
\end{aligned}
$$

It is evident that the recovery voltage is of power frequency and has a negligible transient component. Hence it is better called as power frequency recovery voltage. There is no step jump in the recovery voltage at the instant the breaker opens.

2.1.2 Capacitor Bank with Inrush Reactor - CB-L-C and CB-C-L configurations

The presence of inrush reactors in series with the bank capacitance will cause the bank capacitance voltage to be greater than the peak system voltage at the instant the breaker opens. The capacitor retains this voltage after the breaker opening. The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at $t = 0$ is given by Eq (1.11).

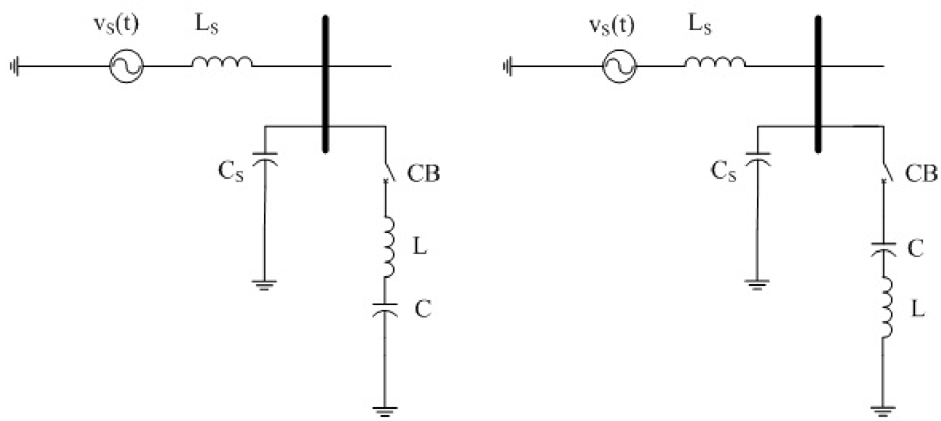

Figure 2.3: Capacitor De-energization with Inrush Reactor - CB-L-C and CB-C-L Configurations

$$v_{C_S}(t) = V_m cos\omega t - [V_m - v_{C_S}(0)]\, cos\omega_0 t \quad \forall t \geq 0 \tag{2.9}$$

Neglecting the voltage drop across the source reactance and using voltage division,

$$v_C(t) = v_C(0) \approx \frac{X_C}{X_C - X_L} V_m \quad \forall t \geq 0 \tag{2.10}$$

$$v_{C_S}(0) \approx V_m \tag{2.11}$$

$$v_{TRV}(t) = v_{C_S}(t) - v_C(t) \tag{2.12}$$

$$v_{TRV}(t) = V_m cos\omega t - \frac{X_C}{X_C - X_L} V_m \tag{2.13}$$

$$v_{TRV}(t) = V_m \left(cos\omega t - \frac{X_C}{X_C - X_L} \right) \quad \forall t \geq 0^+ \tag{2.14}$$

$$v_{TRV}(0^+) = -\frac{X_L}{X_C - X_L} V_m \tag{2.15}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{2.16}$$

$$v_{TRV}(0^-) \neq v_{TRV}(0^+) \tag{2.17}$$

Validation

$$\begin{aligned} L_s &= 5 \ \text{mH} \\ C_s &= 0.29 \ \mu\text{F} \\ L &= 30 \ \text{mH} \\ C &= 8.4239 \ \mu\text{F} \\ V_m &= 93.897 \ \text{kV} \\ \omega &= 376.99 \ \text{rad/sec} \end{aligned}$$

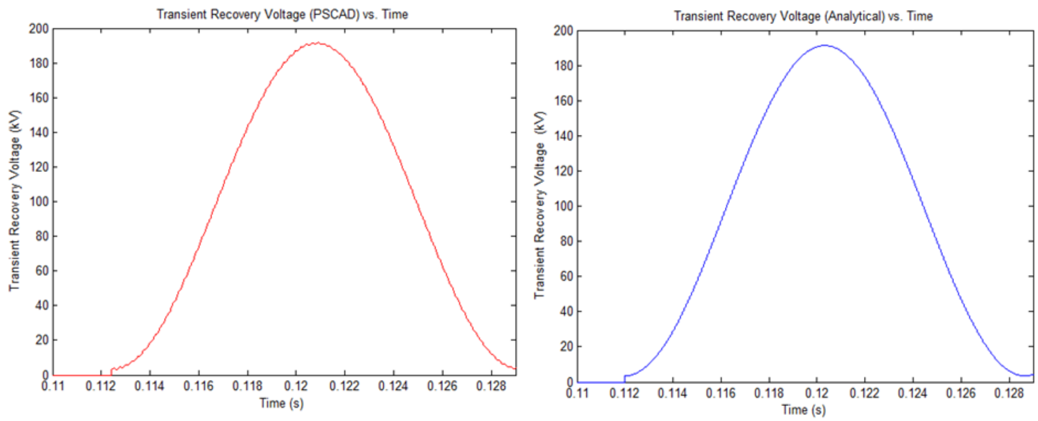

Figure 2.4: PSCAD and MATLAB Validation, Capacitor De-energization with Inrush Reactor - CB-L-C and CB-C-L Configurations

$$
\begin{aligned}
t_{open} &= 0.112 \ \text{sec} \\
\forall t &\geq t_{open}^{+} \\
v_{TRV}(t) &= -V_m \left(cos\omega \left(t - t_{open} \right) - \frac{X_C}{X_C - X_L} \right) \\
v_{TRV}(t_{open}^{+}) &= V_m \left(\frac{X_L}{X_C - X_L} \right) = 3.5 \ \text{kV} \\
v_{TRV,Peak} &= V_m \left(1 + \frac{X_C}{X_C - X_L} \right) = 191.292 \ \text{kV}
\end{aligned}
$$

In both these configurations, the initial voltage condition of the stray capacitor is the same. The voltages held by the bank capacitor after the breaker opening are equal. Hence the recovery voltage seen across the breaker is the same for both the configurations. It is evident that the recovery voltage is of power frequency and has a negligible transient component. Hence it is better called as power frequency recovery voltage. There is a small step jump in the recovery voltage at the instant the breaker opens. This results due to the interruption of the current flowing through the inrush reactor in series with the capacitor.

2.1.3 Capacitor Bank with Inrush Reactor - CB-L||Cp-C Configuration

The presence of the L||Cp element in series with the bank capacitance will cause the bank capacitance voltage to be greater than the peak system voltage at the instant the breaker opens. The capacitor retains this voltage after the breaker opening. A transient voltage appears across the parallel capacitor C_P after the breaker opening. The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at $t = 0$ is given by Eq (1.11).

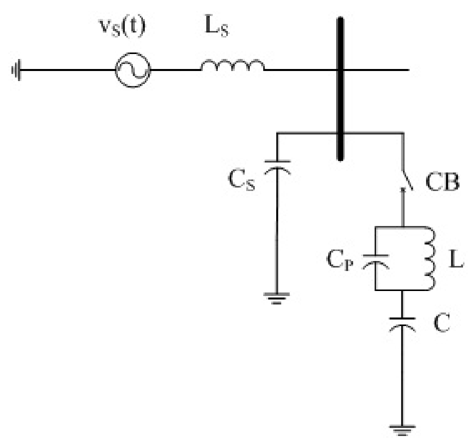

Figure 2.5: Capacitor De-energization with Inrush Reactor - CB-L||Cp-C Configuration

$$v_{C_S}(t) = V_m cos\omega t - \left[V_m - v_{C_S}(0)\right] cos\omega_0 t \quad \forall t \geq 0 \tag{2.18}$$

Neglecting the voltage drop across the source reactance,

$$v_{C_S}(0) \approx V_m \tag{2.19}$$

$$v_{C_S}(t) = V_m cos\omega t \tag{2.20}$$

Define the equivalent inductive reactance of the L||Cp element as

$$X_{eq} = \frac{X_{C_P} X_L}{X_{C_P} - X_L}$$

Using voltage division,

$$v_C(t) = v_C(0) \approx \frac{X_C}{X_C - X_{eq}} V_m \quad \forall t \geq 0 \tag{2.21}$$

The transient voltage that appears across the parallel capacitor after the breaker opening is given by Eq (1.24).

$$v_{C_P}(t) = v_{C_P}(0) cos\omega_P t \quad \forall t \geq 0 \tag{2.22}$$

Using voltage division,

$$v_{C_P}(0) \approx \frac{-X_{eq}}{X_C - X_{eq}} V_m \tag{2.23}$$

$$v_{C_P}(t) = -\frac{X_{eq}}{X_C - X_{eq}} V_m cos\omega_P t \tag{2.24}$$

$$v_{TRV}(t) = v_{C_S}(t) - (v_C(t) + v_{C_P}(t)) \tag{2.25}$$

$$v_{TRV}(t) = V_m cos\omega t - \left(\frac{X_C}{X_C - X_{eq}} V_m - \frac{X_{eq}}{X_C - X_{eq}} V_m cos\omega_P t \right) \tag{2.26}$$

$$v_{TRV}(t) = V_m \left(cos\omega t + \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t - \frac{X_C}{X_C - X_{eq}} \right) \quad \forall t \geq 0^+ \tag{2.27}$$

$$v_{TRV}(0^+) = V_m \left(1 + \frac{X_{eq}}{X_C - X_{eq}} - \frac{X_C}{X_C - X_{eq}} \right) = 0 \tag{2.28}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{2.29}$$

$$v_{TRV}(0^-) = v_{TRV}(0^+) = v_{TRV}(0) = 0 \tag{2.30}$$

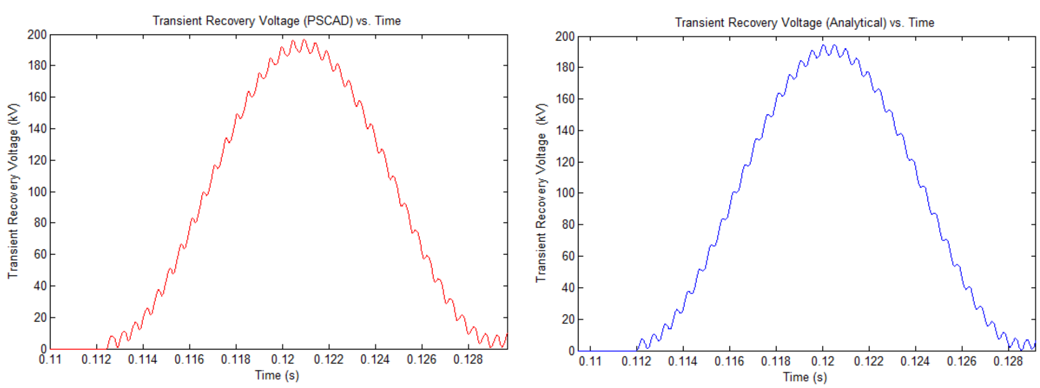

Figure 2.6: PSCAD and MATLAB Validation, Capacitor De-energization with Inrush Reactor - CB-L||Cp-C Configuration

Validation

$$L_s = 5 \ \text{mH}$$
$$C_s = 0.29 \ \mu\text{F}$$
$$L = 30 \ \text{mH}$$
$$C_P = 0.2005 \ \mu\text{F}$$
$$C = 8.4239 \ \mu\text{F}$$
$$V_m = 93.897 \ \text{kV}$$
$$\omega = 376.99 \ \text{rad/sec}$$
$$\omega_P = \frac{1}{\sqrt{LC_P}} = 12893.8 \ \text{rad/sec}$$

$$t_{open} = 0.112 \ \text{sec}$$
$$\forall t \geq t_{open}$$
$$v_{TRV}(t) = -V_m \left(cos\omega(t - t_{open}) + \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P(t - t_{open}) - \frac{X_C}{X_C - X_{eq}} \right)$$
$$v_{TRV}(t_{open}) = -V_m \left(1 + \frac{X_{eq}}{X_C - X_{eq}} - \frac{X_C}{X_C - X_{eq}} \right) = 0$$

The significant component of the recovery voltage is the power frequency but a small transient frequency component is also observed. The small transient component appears due to the oscillation in the L||Cp element. There is no step jump in the recovery voltage at the instant the breaker opens. The parallel capacitor across the inrush reactor prevents a jump in the reactor voltage at the instant the breaker opens.

2.2 TRV Initiated during Fault Clearing

The circuit breaker opens at the current zero to clear the fault which has occurred on the load side of the breaker. The current drawn with the occurrence of the fault can be inductive or capacitive depending on both the configuration of the capacitor bank with inrush reactors and the relative location of the fault on the load side. The source voltage is assumed to be $v_S(t) = V_m cos\omega t$. Hence, when the breaker opens at $t = 0$, the source voltage is at the positive maximum V_m while the current flowing through it is zero. The fault is assumed to be a bolted fault and the voltage at the fault point is taken to be zero or is at ground potential.

2.2.1 Fault at Load Side CB Terminal - All Capacitor Configurations

The circuit is predominantly inductive as the load side is shorted out by the fault. The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at $t = 0$ is given by Eq (1.11).

$$v_{C_S}(t) = V_m cos\omega t - [V_m - v_{C_S}(0)]\, cos\omega_0 t \quad \forall t \geq 0 \tag{2.31}$$

The fault is a bolted fault and hence the fault point potential is at the ground potential.

$$v_{Fault}(t) = 0 \tag{2.32}$$

$$v_{C_S}(0) = 0 \tag{2.33}$$

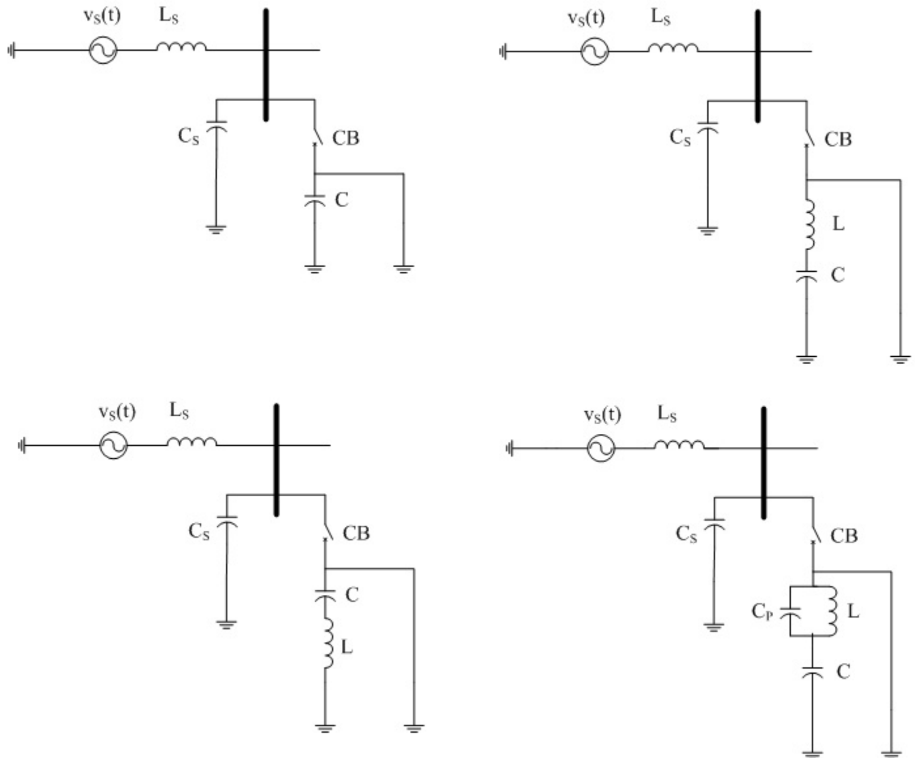

Figure 2.7: Fault at Load Side CB Terminal - All Capacitor Configurations

$$v_{TRV}(t) = v_{C_S}(t) - v_{Fault}(t) \tag{2.34}$$

$$v_{TRV}(t) = V_m cos\omega t - V_m cos\omega_0 t \tag{2.35}$$

$$v_{TRV}(t) \approx V_m \left(1 - cos\omega_0 t\right) \quad \forall t \geq 0^+ \tag{2.36}$$

$$v_{TRV}(0^{+}) = 0 \tag{2.37}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^{-}) = 0 \tag{2.38}$$

$$v_{TRV}(0^{-}) = v_{TRV}(0^{+}) = 0 \tag{2.39}$$

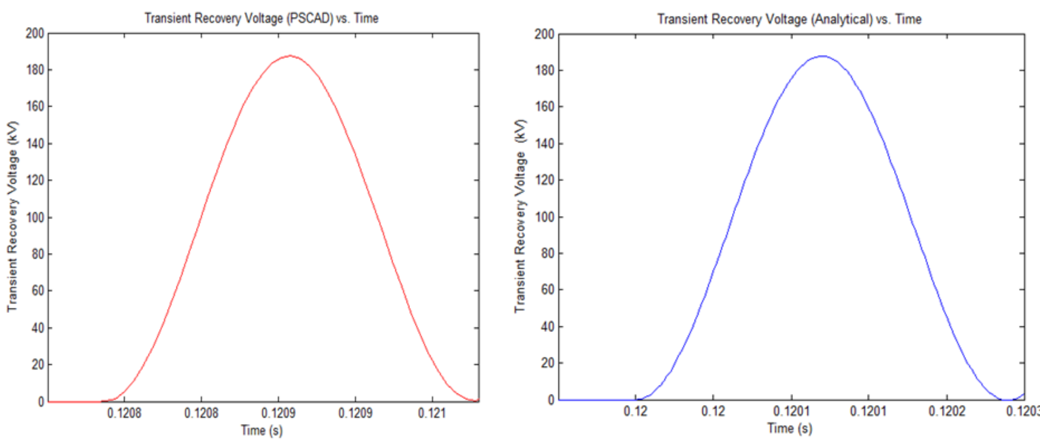

Figure 2.8: PSCAD and MATLAB Validation, Fault at Load Side CB Terminal - All Capacitor Configurations

Validation

$$L_s = 5 \;\; \text{mH}$$

$$C_s = 0.29 \;\; \mu\text{F}$$

$$L = 30 \;\; \text{mH}$$

$$C_P = 0.2005 \;\; \mu\text{F}$$

$$C = 8.4239 \;\; \mu\text{F}$$

$$V_m = 93.897 \;\; \text{kV}$$

$$\omega = 376.99 \;\; \text{rad/sec}$$

$$\omega_0 = \frac{1}{\sqrt{L_S C_S}} = 26261 \;\; \text{rad/sec}$$

$$t_{open} = 0.12 \;\; \text{sec}$$

$$\forall t \geq t_{open}$$

$$v_{TRV}(t) \approx V_m \left(1 - cos\omega_0 \left(t - t_{open}\right)\right)$$

$$v_{TRV}(t_{open}) = 0$$

$$v_{TRV,Peak} = 2V_m = 187.794 \;\; \text{kV}$$

The recovery voltage constitutes a high frequency component and a power frequency component, but the power frequency term is assumed to be a constant for the small transient time span of interest. There is no step jump in the TRV at the instant the breaker opens. The stray capacitance at the breaker terminal has a zero initial condition at time t_{open}.

2.2.2 CB-L-C Configuration - Fault between L and C Terminals

The circuit is predominantly inductive as the capacitor is shorted out by the fault. The fault current flows through the inrush reactor and when the breaker opens, the current through the inrush reactor is interrupted. The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at $t = 0$ is given by Eq (1.11).

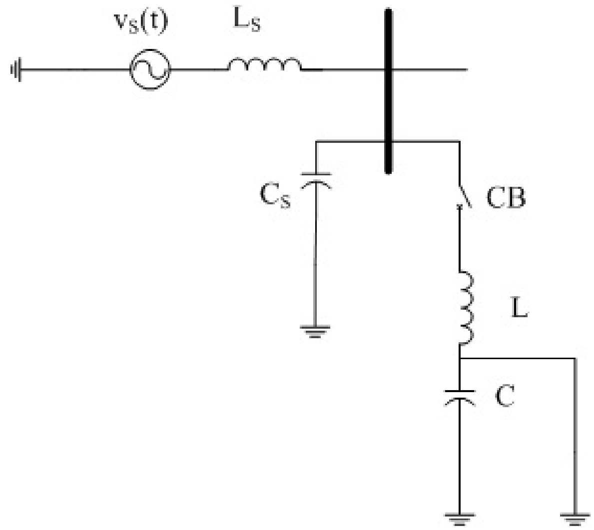

Figure 2.9: CB-L-C configuration- Fault between L and C Terminals

$$v_{C_S}(t) = V_m cos\omega t - [V_m - v_{C_S}(0)]\, cos\omega_0 t \quad \forall t \geq 0 \tag{2.40}$$

The fault is a bolted fault and hence the fault point potential is at the ground potential.

$$v_{Fault}(t) = 0 \tag{2.41}$$

Using voltage division,

$$v_{C_S}(0) = \frac{L}{L + L_S} V_m \tag{2.42}$$

$$v_{TRV}(t) = v_{C_S}(t) - v_{Fault}(t) \tag{2.43}$$

$$v_{TRV}(t) = V_m cos\omega t - \frac{L_S}{L + L_S} V_m cos\omega_0 t \tag{2.44}$$

$$v_{TRV}(t) \approx V_m \left(1 - \frac{L_S}{L + L_S} cos\omega_0 t\right) \quad \forall t \geq 0^+ \tag{2.45}$$

$$v_{TRV}(0^+) = \frac{L}{L + L_S} V_m \tag{2.46}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{2.47}$$

$$v_{TRV}(0^-) \neq v_{TRV}(0^+) \tag{2.48}$$

Validation

$$\begin{aligned}
L_s &= 5 \text{ mH} \\
C_s &= 0.29 \ \mu\text{F} \\
L &= 30 \text{ mH} \\
C &= 8.4239 \ \mu\text{F} \\
V_m &= 93.897 \text{ kV} \\
\omega &= 376.99 \text{ rad/sec} \\
\omega_0 &= \frac{1}{\sqrt{L_S C_S}} = 26261 \text{ rad/sec}
\end{aligned}$$

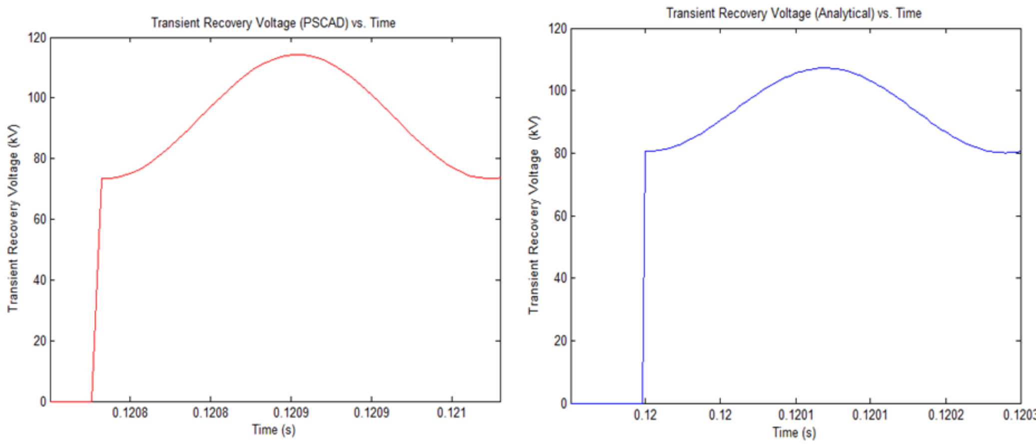

Figure 2.10: PSCAD and MATLAB Validation, CB-L-C Configuration - Fault between L and C Terminals

$$
\begin{aligned}
t_{open} &= 0.12 \ \text{sec} \\
\forall t &\geq t_{open}^{+} \\
v_{TRV}(t) &\approx V_m \left(1 - \frac{L_S}{L + L_S} cos\omega_0 \left(t - t_{open}\right)\right) \\
v_{TRV}(t_{open}^{+}) &= V_m \left(\frac{L}{L + L_S}\right) = 80.483 \ \text{kV} \\
v_{TRV,Peak} &= V_m \left(1 + \frac{L_S}{L + L_S}\right) = 107.311 \ \text{kV}
\end{aligned}
$$

The recovery voltage constitutes a high frequency component and a power frequency component, but the power frequency term is assumed to be a constant for the small transient time span of interest. There is a significant step jump in the TRV at the instant the breaker opens. The stray capacitance at the breaker terminal has a non-zero initial condition at time t_{open}. The breaker interrupts the current flowing through the inrush reactor when it opens.

2.2.3 CB-C-L Configuration - Fault between C and L Terminals

The circuit is predominantly capacitive as only the inrush reactor is shorted out by the fault. The breaker interrupts capacitive current when it opens. This case is similar to capacitor de-energization without inrush reactors. The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at $t = 0$ is given by Eq (1.11).

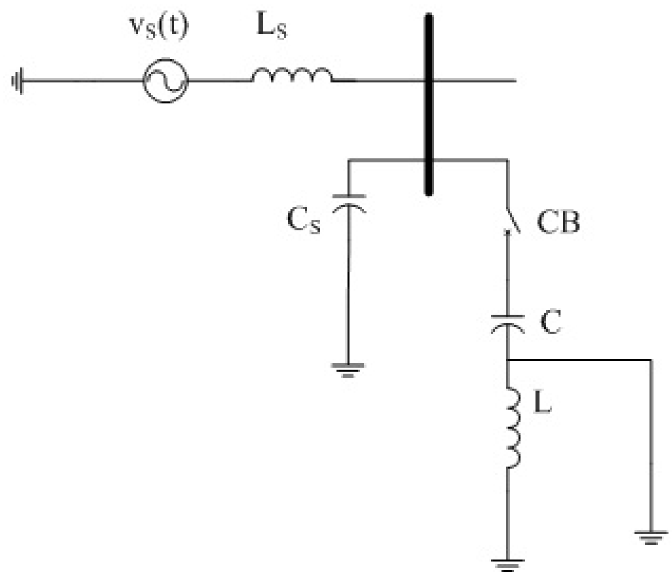

Figure 2.11: CB-C-L configuration - Fault between C and L Terminals

$$v_{C_S}(t) = V_m cos\omega t - [V_m - v_{C_S}(0)]\, cos\omega_0 t \quad \forall t \geq 0 \tag{2.49}$$

Neglecting the voltage drop across the source reactance, the voltage at the bus is the peak voltage at current zero. The capacitor bank retains this voltage after the opening of the circuit breaker.

$$v_C(t) = v_C(0) = v_{C_S}(0) \approx V_m \quad \forall t \geq 0 \tag{2.50}$$

$$v_{TRV}(t) = v_{C_S}(t) - v_C(t) \tag{2.51}$$

$$v_{TRV}(t) = V_m cos\omega t - V_m \tag{2.52}$$

$$v_{TRV}(t) = V_m \left(cos\omega t - 1\right) \quad \forall t \geq 0^+ \tag{2.53}$$

$$v_{TRV}(0^+) = 0 \tag{2.54}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{2.55}$$

$$v_{TRV}(0^-) = v_{TRV}(0^+) = 0 \tag{2.56}$$

Validation

$$\begin{aligned} L_s &= 5 \ \text{mH} \\ C_s &= 0.29 \ \mu\text{F} \\ L &= 30 \ \text{mH} \\ C &= 8.4239 \ \mu\text{F} \\ V_m &= 93.897 \ \text{kV} \\ \omega &= 376.99 \ \text{rad/sec} \end{aligned}$$

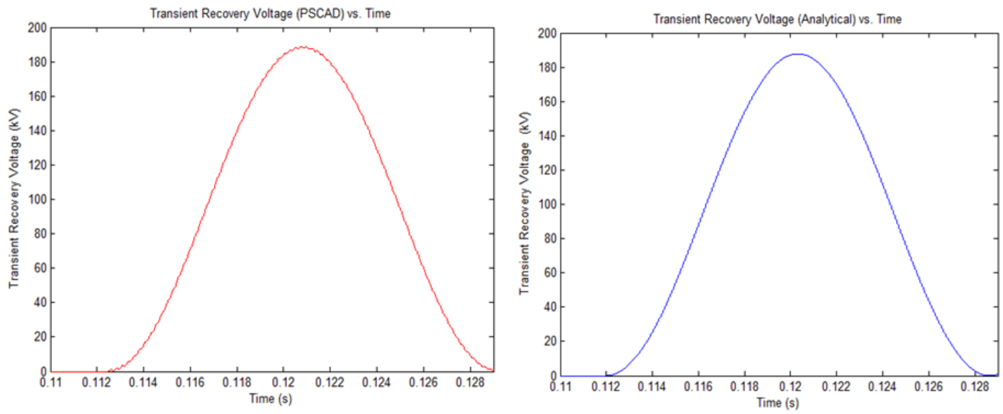

Figure 2.12: PSCAD and MATLAB Validation, CB-C-L Configuration - Fault between C and L Terminals

$$
\begin{aligned}
t_{open} &= 0.112 \ \text{sec} \\
\forall t &\geq t_{open} \\
v_{TRV}(t) &= -V_m \left(cos\omega \left(t - t_{open} \right) - 1 \right) \\
v_{TRV}(t_{open}) &= 0 \\
v_{TRV,Peak} &= 2V_m = 187.794 \ \text{kV}
\end{aligned}
$$

It is evident that the recovery voltage is of power frequency and has a negligible transient component. Hence it is better called as power frequency recovery voltage. There is no step jump in the recovery voltage at the instant the breaker opens.

2.2.4 CB-L||Cp-C Configuration - Fault between L||Cp and C Terminals

The circuit is predominantly inductive as the capacitor is shorted out by the fault and the L||Cp element has a net inductive reactance. But the current through the inductor is not interrupted due to the parallel capacitance. The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at $t = 0$ is given by Eq (1.11).

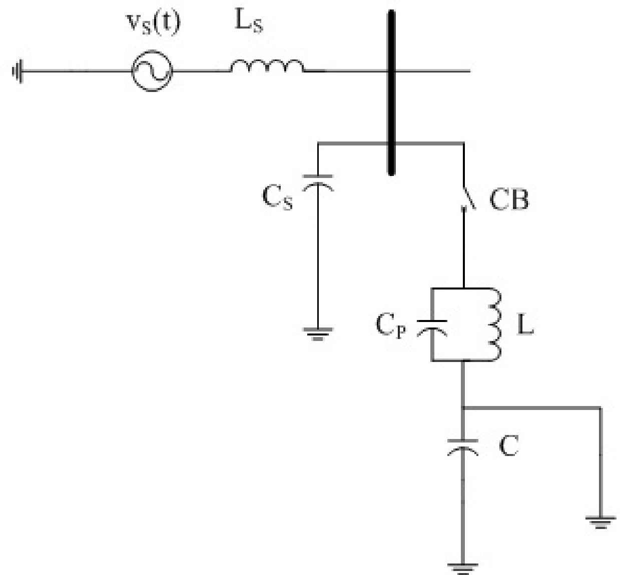

Figure 2.13: CB-L||Cp-C Configuration - Fault between L||Cp and C Terminals

$$v_{C_S}(t) = V_m cos\omega t - [V_m - v_{C_S}(0)]\, cos\omega_0 t \quad \forall t \geq 0 \tag{2.57}$$

Define the equivalent inductive reactance of the L||Cp element as

$$X_{eq} = \frac{X_{C_P} X_L}{X_{C_P} - X_L}$$

Using voltage division,

$$v_{C_S}(0) = \frac{X_{eq}}{X_{eq} + X_{L_S}} V_m \tag{2.58}$$

$$v_{C_S}(t) = V_m cos\omega t - \left[V_m - \frac{X_{eq}}{X_{eq} + X_{L_S}} V_m \right] cos\omega_0 t \tag{2.59}$$

$$v_{C_S}(t) = V_m cos\omega t - V_m \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0 t \tag{2.60}$$

The transient voltage that appears across the parallel capacitor after the breaker opening is given by Eq (1.24).

$$v_{C_P}(t) = v_{C_P}(0) cos\omega_P t \quad \forall t \geq 0 \tag{2.61}$$

Using voltage division,

$$v_{C_P}(0) = \frac{X_{eq}}{X_{eq} + X_{L_S}} V_m \tag{2.62}$$

$$v_{C_P}(t) = \frac{X_{eq}}{X_{eq} + X_{L_S}} V_m cos\omega_P t \tag{2.63}$$

The fault is a bolted fault and hence the fault point potential is at the ground potential.

$$v_{Fault}(t) = 0 \tag{2.64}$$

$$v_{TRV}(t) = v_{C_S}(t) - (v_{C_P}(t) + v_{Fault}(t)) \tag{2.65}$$

$$v_{TRV}(t) = V_m cos\omega t - V_m \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0 t - \left(\frac{X_{eq}}{X_{eq} + X_{L_S}} V_m cos\omega_P t \right) \quad (2.66)$$

$$v_{TRV}(t) \approx V_m \left(1 - \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0 t - \frac{X_{eq}}{X_{eq} + X_{L_S}} cos\omega_P t \right) \qquad \forall t \geq 0^+ \quad (2.67)$$

$$v_{TRV}(0^+) = V_m \left(1 - \frac{X_{L_S}}{X_{eq} + X_{L_S}} - \frac{X_{eq}}{X_{eq} + X_{L_S}} \right) = 0 \quad (2.68)$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \quad (2.69)$$

$$v_{TRV}(0^-) = v_{TRV}(0^+) = 0 \quad (2.70)$$

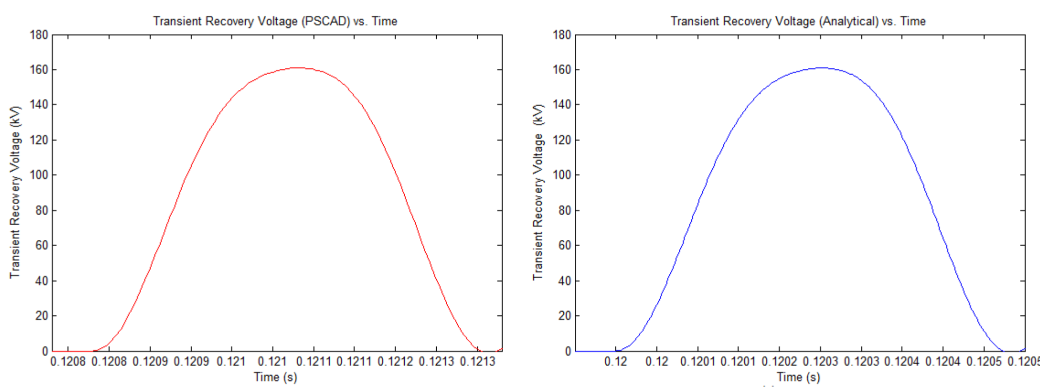

Figure 2.14: PSCAD and MATLAB Validation, CB-L||Cp-C configuration - Fault between L||Cp and C Terminals

Validation

$$L_s = 5 \ \text{mH}$$

$$C_s = 0.29 \ \mu\text{F}$$

$$L = 30 \ \text{mH}$$

$$C_P = 0.2005 \ \mu\text{F}$$

$$C = 8.4239 \ \mu\text{F}$$

$$V_m = 93.897 \ \text{kV}$$

$$\omega = 376.99 \ \text{rad/sec}$$

$$\omega_0 = \frac{1}{\sqrt{L_S C_S}} = 26261 \ \text{rad/sec}$$

$$\omega_P = \frac{1}{\sqrt{L C_P}} = 12893.8 \ \text{rad/sec}$$

$$t_{open} = 0.12 \ \text{sec}$$

$$\forall t \geq t_{open}$$

$$v_{TRV}(t) = V_m \left(1 - \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0(t - t_{open}) - \frac{X_{eq}}{X_{eq} + X_{L_S}} cos\omega_P(t - t_{open})\right)$$

$$v_{TRV}(t_{open}) = V_m \left(1 - \frac{X_{L_S}}{X_{eq} + X_{L_S}} - \frac{X_{eq}}{X_{eq} + L_S}\right) = 0$$

The recovery voltage constitutes two high frequency components and a power frequency component, but the power frequency term is assumed to be a constant for the small transient time span of interest. The waveform constitutes both the high frequency components, but only the ω_P component is evident. ω_0 is about twice ω_P. The ω_0 component adds to the ω_P component at the rise and opposes at the peak. Hence the peak magnitude is less and the waveform is flat close to the peak. There is no step jump in the TRV at the instant the breaker opens. This is because of the parallel capacitor across the inrush reactor.

The recovery voltage analysis for a three-phase grounded capacitor bank with a three-phase fault to ground is similar to the single-phase analysis that has been presented in this chapter. The recovery voltage across the first breaker pole to open is the same as the recovery voltage appearing across the circuit breaker in the corresponding single-phase configuration. The next chapter presents the TRV analysis for three-phase ungrounded capacitor bank configurations which behave differently compared to the single-phase and the three-phase grounded capacitor bank configurations.

Chapter 3

Three-phase Ungrounded Capacitor Banks

The transient recovery voltage analysis for three-phase ungrounded capacitor bank configurations has been presented in this chapter. The recovery voltage analysis is first carried out for de-energization of the various capacitor bank - inrush reactor configurations. This is followed by the analysis of the TRV initiated when the first pole of the circuit breaker opens to clear the fault. The fault is a three-phase to neutral ungrounded fault. The fault is assumed to be a bolted fault and hence the fault resistance is zero. Analysis has been presented for faults occurring at various possible locations of the different capacitor bank configurations. The derived results have been supported by PSCAD and MATLAB simulations.

3.1 De-energization of Capacitor Bank

The first pole of the circuit breaker, say phase A opens at the current zero to de-energize the capacitor on phase A. The load is predominantly capacitive and hence, before the opening of the breaker, the line current approximately leads the source voltage by 90°. The source voltage of phase A is assumed to be $v_a(t) = V_m cos\omega t$. Hence when the breaker opens at $t = 0$, voltage of phase A is at the positive maximum V_m while the current is at the zero crossing, where current goes from positive to negative. At this instant the source voltages of phase B and C are $-\frac{V_m}{2}$.

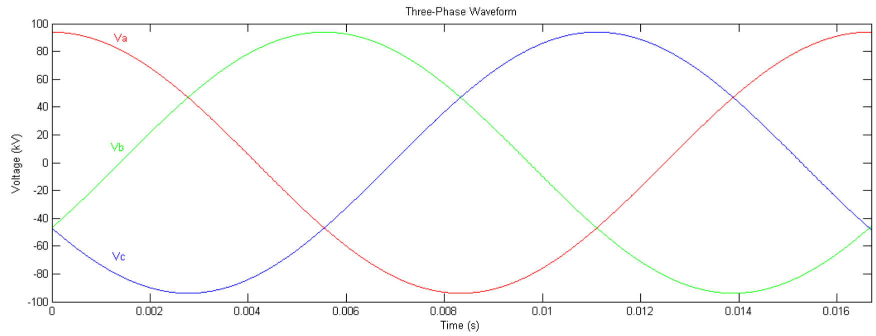

Figure 3.1: Three-Phase Voltage Waveform

After the opening of the first pole, there is asymmetry in the circuit and the neutral is no more at the ground potential. Hence calculations involve estimating the neutral displacement potential followed by estimation of the recovery voltage. The de-energization analysis of a 3-phase ungrounded capacitor bank with no inrush reactors and with inrush reactors is presented in the following sections.

3.1.1 Capacitor Bank without Inrush Reactors

For de-energization analysis, there is negligible current flowing through the stray capacitances even after opening of the first pole of the circuit breaker. The current flows only through the main bank capacitors.

Estimation of Neutral Point Voltage for De-energization

From Kirchoff's Voltage Law (KVL) applied to phase B and phase C circuits,

$$v_b(t) = L_S \frac{di_b(t)}{dt} + v_{C_b}(t) + v_n(t) \tag{3.1}$$

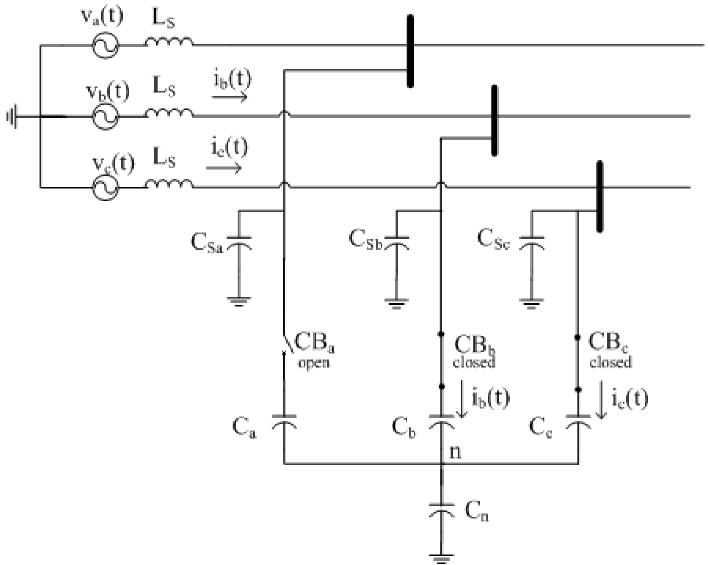

Figure 3.2: De-energization of 3-Phase Capacitor Bank without Inrush Reactors

$$v_c(t) = L_S \frac{di_c(t)}{dt} + v_{C_c}(t) + v_n(t) \tag{3.2}$$

Adding Eq (3.1) and Eq (3.2),

$$v_b(t) + v_c(t) = L_S \frac{di_b(t)}{dt} + L_S \frac{di_c(t)}{dt} + v_{C_b}(t) + v_{C_c}(t) + 2v_n(t) \tag{3.3}$$

$$v_b(t) + v_c(t) = L_S \frac{d\left(i_b(t) + i_c(t)\right)}{dt} + v_{C_b}(t) + v_{C_c}(t) + 2v_n(t) \tag{3.4}$$

The breaker pole of phase A opens at $t = 0$ and hence no phase A line current flows to the neutral. From Kirchoff's Current Law (KCL) applied at the neutral n,

$$i_b(t) + i_c(t) = 0 \quad \forall t \geq 0 \tag{3.5}$$

For a balanced three-phase supply, the sum of the three phase voltages are zero at any instant of time and hence,

$$v_b(t) + v_c(t) = -v_a(t) \tag{3.6}$$

Using Eq (3.5) and Eq (3.6) in Eq (3.4),

$$-v_a(t) = v_{C_b}(t) + v_{C_c}(t) + 2v_n(t) \tag{3.7}$$

The voltage across the phase B and phase C capacitors are given by,

$$v_{C_b}(t) = \frac{1}{C}\int_{-\infty}^{t} i_b(t)dt = v_{C_b}(0) + \frac{1}{C}\int_{0}^{t} i_b(t)dt \tag{3.8}$$

$$v_{C_c}(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt = v_{C_c}(0) + \frac{1}{C}\int_{0}^{t} i_c(t)dt \tag{3.9}$$

The integral from negative infinity to zero yields the initial condition and the integral from zero to time t gives the general time varying expression.
Adding Eq (3.8) and Eq (3.9),

$$v_{C_b}(t) + v_{C_c}(t) = v_{C_b}(0) + v_{C_c}(0) + \frac{1}{C}\int_{0}^{t} \left(i_b(t) + i_c(t)\right) dt \tag{3.10}$$

Using Eq (3.5) in Eq (3.10) and applying the initial conditions for the phase B and phase C capacitors obtained by neglecting the voltage drop across the source reactors,

$$v_{C_b}(t) + v_{C_c}(t) = v_{C_b}(0) + v_{C_c}(0) \approx \frac{-V_m}{2} + \frac{-V_m}{2} \tag{3.11}$$

$$v_{C_b}(t) + v_{C_c}(t) \approx -V_m \tag{3.12}$$

Substituting Eq (3.12) in Eq (3.7),

$$-v_a(t) = -V_m + 2v_n(t) \tag{3.13}$$

$$-V_m cos\omega t = -V_m + 2v_n(t) \tag{3.14}$$

$$v_n(t) = \frac{V_m}{2}\left(1 - cos\omega t\right) \quad \forall t \geq 0^+ \tag{3.15}$$

Recovery Voltage

The recovery voltage developed across the first breaker pole to open is estimated by analyzing the voltages on the source and load side of the breaker pole of interest. The circuit on the source side of the pole is a single-phase circuit. Hence the single-phase equation for the voltage developed across the stray capacitance at the source side terminal of the breaker after the opening of the circuit breaker is applicable. The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at time $t = 0$ is given by Eq (1.11). This equation is applied to the phase A circuit.

$$v_{C_{Sa}}(t) = V_m cos\omega t - \left[V_m - v_{C_{Sa}}(0)\right] cos\omega_0 t \quad \forall t \geq 0 \tag{3.16}$$

Neglecting the voltage drop in the source reactance,

$$v_{C_a}(t) = v_{C_a}(0) = v_{C_{Sa}}(0) \approx V_m \quad \forall t \geq 0 \tag{3.17}$$

$$v_{TRV}(t) = v_{C_{Sa}}(t) - \left(v_{C_a}(t) + v_n(t)\right) \tag{3.18}$$

$$v_{TRV}(t) = V_m cos\omega t - \left(V_m + \frac{V_m}{2}(1 - cos\omega t) \right) \tag{3.19}$$

$$v_{TRV}(t) = \frac{3}{2} V_m (cos\omega t - 1) \quad \forall t \geq 0^+ \tag{3.20}$$

$$v_{TRV}(0^+) = 0 \tag{3.21}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{3.22}$$

$$v_{TRV}(0^-) = v_{TRV}(0^+) = 0 \tag{3.23}$$

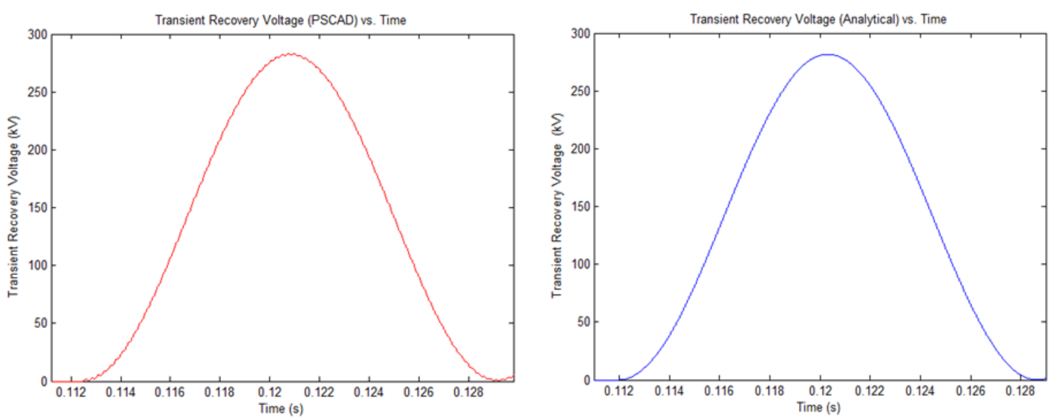

Figure 3.3: PSCAD and MATLAB Validation, De-energization of 3-Phase Capacitor Bank without Inrush Reactors

Validation

$$
\begin{aligned}
L_s &= 5 \ \text{mH} \\
C_s &= 0.29 \ \mu\text{F} \\
C &= 8.4239 \ \mu\text{F} \\
V_m &= 93.897 \ \text{kV} \\
\omega &= 376.99 \ \text{rad/sec}
\end{aligned}
$$

$$
\begin{aligned}
t_{open} &= 0.112 \ \text{sec} \\
\forall t &\geq t_{open} \\
v_{TRV}(t) &= -\frac{3}{2}V_m \left(cos\omega \left(t - t_{open}\right) - 1\right) \\
v_{TRV}(t_{open}) &= 0 \\
v_{TRV,Peak} &= 3V_m = 281.691 \ \text{kV}
\end{aligned}
$$

It is evident that the recovery voltage is of power frequency and has a negligible transient component. Hence it is better called as power frequency recovery voltage. There is no step jump in the recovery voltage at the instant the breaker pole of phase A opens. But the recovery voltage is 1.5 times greater than that of the corresponding single-phase capacitor de-energization.

3.1.2 Capacitor Bank with Inrush Reactors - CB-C-L and CB-L-C Configurations

For de-energization analysis, there is negligible current flowing through the stray capacitances even after opening of the first pole of the circuit breaker. The current flows only through the main bank capacitors. The neutral point voltage is not at ground potential after the opening of the first pole of the breaker and is estimated next.

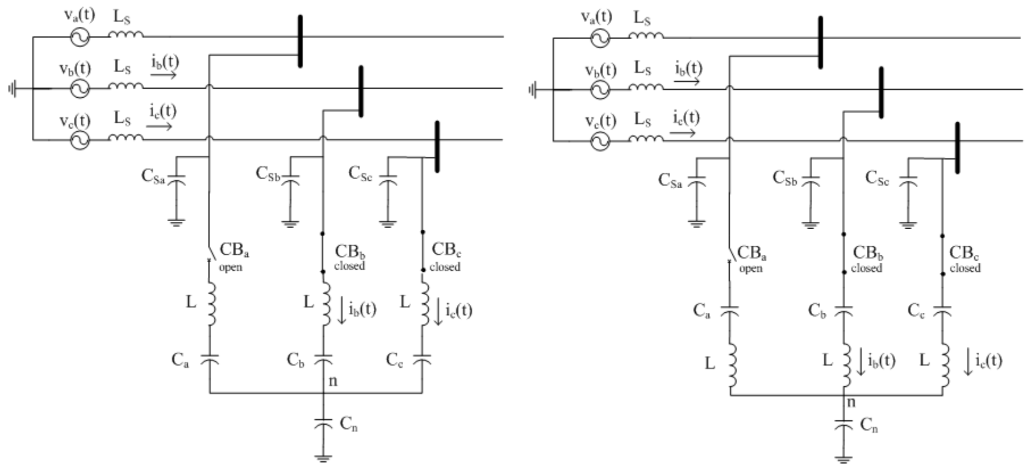

Figure 3.4: De-energization of 3-Phase Capacitor Bank with Inrush Reactors - CB-C-L and CB-L-C Configurations

Estimation of Neutral Point Voltage for De-energization

From KVL applied to phase B and phase C circuits,

$$v_b(t) = L_S \frac{di_b(t)}{dt} + L \frac{di_b(t)}{dt} + v_{C_b}(t) + v_n(t) \tag{3.24}$$

$$v_c(t) = L_S \frac{di_c(t)}{dt} + L \frac{di_c(t)}{dt} + v_{C_c}(t) + v_n(t) \tag{3.25}$$

Adding Eq (3.24) and Eq (3.25),

$$\begin{aligned} v_b(t) + v_c(t) = &(L_S + L)\frac{di_b(t)}{dt} + (L_S + L)\frac{di_c(t)}{dt} \\ &+ v_{C_b}(t) + v_{C_c}(t) + 2v_n(t) \end{aligned} \tag{3.26}$$

$$\begin{aligned} v_b(t) + v_c(t) = &(L_S + L)\frac{d\left(i_b(t) + i_c(t)\right)}{dt} \\ &+ v_{C_b}(t) + v_{C_c}(t) + 2v_n(t) \end{aligned} \tag{3.27}$$

The breaker pole of phase A opens at $t = 0$ and hence no phase A line current flows to the neutral. From KCL applied at the neutral n,

$$i_b(t) + i_c(t) = 0 \quad \forall t \geq 0 \tag{3.28}$$

For a balanced three-phase supply, the sum of the three phase voltages are zero at any instant of time and hence,

$$v_b(t) + v_c(t) = -v_a(t) \tag{3.29}$$

Using Eq (3.28) and Eq (3.29) in Eq (3.27),

$$-v_a(t) = v_{C_b}(t) + v_{C_c}(t) + 2v_n(t) \tag{3.30}$$

The voltage across the phase B and phase C capacitors are given by,

$$v_{C_b}(t) = \frac{1}{C}\int_{-\infty}^{t} i_b(t)dt = v_{C_b}(0) + \frac{1}{C}\int_{0}^{t} i_b(t)dt \tag{3.31}$$

$$v_{C_c}(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt = v_{C_c}(0) + \frac{1}{C}\int_{0}^{t} i_c(t)dt \tag{3.32}$$

Adding Eq (3.31) and Eq (3.32),

$$v_{C_b}(t) + v_{C_c}(t) = v_{C_b}(0) + v_{C_c}(0) + \frac{1}{C}\int_{0}^{t} \left(i_b(t) + i_c(t)\right) dt \tag{3.33}$$

Using Eq (3.28) in Eq (3.33) and applying the initial conditions for the phase B and phase C capacitors obtained by neglecting the voltage drop across the source reactors and using voltage division,

$$v_{C_b}(t) + v_{C_c}(t) = v_{C_b}(0) + v_{C_c}(0) \approx \frac{-V_m}{2}\frac{X_C}{X_C - X_L} + \frac{-V_m}{2}\frac{X_C}{X_C - X_L} \quad (3.34)$$

$$v_{C_b}(t) + v_{C_c}(t) \approx -V_m\frac{X_C}{X_C - X_L} \quad (3.35)$$

Substituting Eq (3.35) in Eq (3.30),

$$-v_a(t) = -V_m\frac{X_C}{X_C - X_L} + 2v_n(t) \quad (3.36)$$

$$-V_m cos\omega t = -V_m\frac{X_C}{X_C - X_L} + 2v_n(t) \quad (3.37)$$

$$v_n(t) = \frac{V_m}{2}\left(\frac{X_C}{X_C - X_L} - cos\omega t\right) \quad \forall t \geq 0^+ \quad (3.38)$$

Recovery Voltage

The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at time $t = 0$ is given by Eq (1.11). This equation is applied to the phase A circuit.

$$v_{C_{Sa}}(t) = V_m cos\omega t - \left[V_m - v_{C_{Sa}}(0)\right] cos\omega_0 t \quad \forall t \geq 0 \quad (3.39)$$

Neglecting the voltage drop in the source reactance,

$$v_{C_{Sa}}(0) \approx V_m \quad (3.40)$$

Using voltage division,

$$v_{C_a}(t) = v_{C_a}(0) \approx V_m \frac{X_C}{X_C - X_L} \quad \forall t \geq 0 \tag{3.41}$$

$$v_{TRV}(t) = v_{C_{Sa}}(t) - (v_{C_a}(t) + v_n(t)) \tag{3.42}$$

$$v_{TRV}(t) = V_m cos\omega t - \left(V_m \left(\frac{X_C}{X_C - X_L} \right) + \frac{V_m}{2} \left(\frac{X_C}{X_C - X_L} - cos\omega t \right) \right) \tag{3.43}$$

$$v_{TRV}(t) = \frac{3}{2} V_m \left(cos\omega t - \frac{X_C}{X_C - X_L} \right) \quad \forall t \geq 0^+ \tag{3.44}$$

$$v_{TRV}(0^+) = \frac{3}{2} V_m \left(1 - \frac{X_C}{X_C - X_L} \right) \tag{3.45}$$

$$v_{TRV}(0^+) = -\frac{3}{2} V_m \left(\frac{X_L}{X_C - X_L} \right) \tag{3.46}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{3.47}$$

$$v_{TRV}(0^-) \neq v_{TRV}(0^+) \tag{3.48}$$

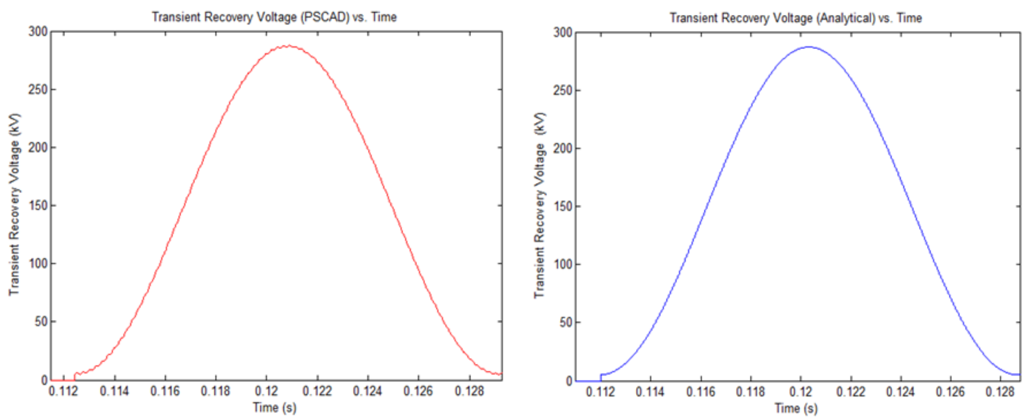

Figure 3.5: PSCAD and MATLAB Validation, De-energization of 3-Phase Capacitor Bank with Inrush Reactors - CB-C-L and CB-L-C Configurations

Validation

$$L_s = 5 \ \text{mH}$$

$$C_s = 0.29 \ \mu\text{F}$$

$$L = 30 \ \text{mH}$$

$$C = 8.4239 \ \mu\text{F}$$

$$V_m = 93.897 \ \text{kV}$$

$$\omega = 376.99 \ \text{rad/sec}$$

$$t_{open} = 0.112 \ \text{sec}$$

$$\forall t \geq t_{open}^{+}$$

$$v_{TRV}(t) = -\frac{3}{2}V_m \left(cos\omega\left(t - t_{open}\right) - \frac{X_C}{X_C - X_L} \right)$$

$$v_{TRV}(t_{open}^{+}) = \frac{3}{2}V_m \left(\frac{X_L}{X_C - X_L} \right) = 5.247 \ \text{kV}$$

$$v_{TRV,Peak} = \frac{3}{2}V_m \left(1 + \frac{X_C}{X_C - X_L} \right) = 286.938 \ \text{kV}$$

It is evident that the recovery voltage is of power frequency and has a negligible transient component. Hence it is better called as power frequency recovery voltage. There is a small step jump in the recovery voltage at the instant the breaker pole of phase A opens. This results due to the interruption of the current flowing through the inrush reactor in series with the capacitor. There is also a small step jump in the neutral potential at the switching instant which causes the step jump in the recovery voltage to be 1.5 times more as compared to the corresponding single-phase case. The recovery voltage is 1.5 times greater than that of the corresponding single-phase capacitor de-energization.

3.1.3 Capacitor Bank with Inrush Reactors - CB-L||Cp-C Configuration

For de-energization analysis, there is negligible current flowing through the stray capacitances even after opening of the first pole of the circuit breaker. The current flows only through the main bank capacitors.

Estimation of Neutral Point Voltage for De-energization

From KVL applied to phase B and phase C circuits,

$$v_b(t) = L_S \frac{di_b(t)}{dt} + v_{C_{Pb}}(t) + v_{C_b}(t) + v_n(t) \tag{3.49}$$

$$v_c(t) = L_S \frac{di_c(t)}{dt} + v_{C_{Pc}}(t) + v_{C_c}(t) + v_n(t) \tag{3.50}$$

Adding Eq (3.49) and Eq (3.50),

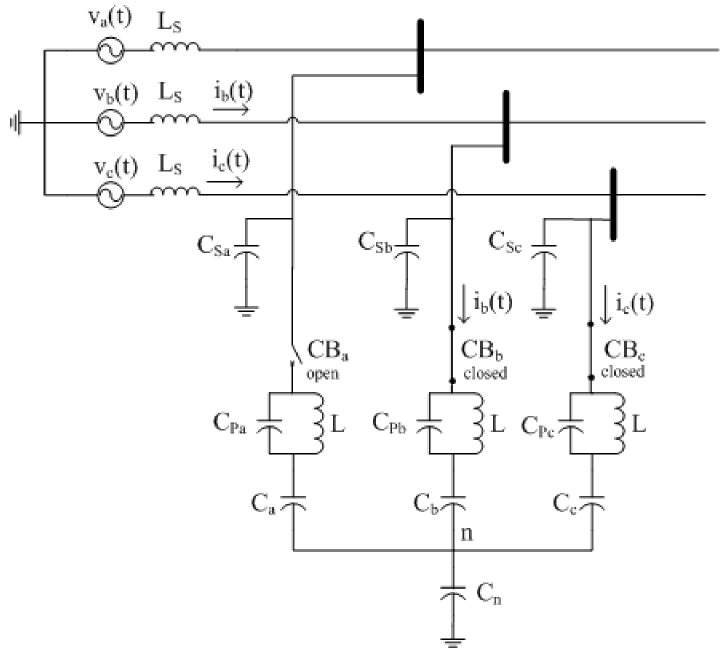

Figure 3.6: De-energization of 3-Phase Capacitor Bank with Inrush Reactors - CB-L||Cp-C Configuration

$$\begin{aligned} v_b(t) + v_c(t) = & L_S \frac{d\left(i_b(t) + i_c(t)\right)}{dt} + \left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right) \\ & + \left(v_{C_b}(t) + v_{C_c}(t)\right) + 2v_n(t) \end{aligned} \tag{3.51}$$

The breaker pole of phase A opens at $t = 0$ and hence no phase A line current flows to the neutral. From KCL applied at the neutral n,

$$i_b(t) + i_c(t) = 0 \quad \forall t \geq 0 \tag{3.52}$$

For a balanced three-phase supply, the sum of the three phase voltages are zero at any instant of time and hence,

$$v_b(t) + v_c(t) = -v_a(t) \tag{3.53}$$

Using Eq (3.52) and Eq (3.53) in Eq (3.51),

$$-v_a(t) = (v_{C_b}(t) + v_{C_c}(t)) + (v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) + 2v_n(t) \tag{3.54}$$

The voltage across the phase B and phase C capacitors are given by,

$$v_{C_b}(t) = \frac{1}{C}\int_{-\infty}^{t} i_b(t)dt = v_{C_b}(0) + \frac{1}{C}\int_{0}^{t} i_b(t)dt \tag{3.55}$$

$$v_{C_c}(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt = v_{C_c}(0) + \frac{1}{C}\int_{0}^{t} i_c(t)dt \tag{3.56}$$

Adding Eq (3.55) and Eq (3.56),

$$v_{C_b}(t) + v_{C_c}(t) = v_{C_b}(0) + v_{C_c}(0) + \frac{1}{C}\int_{0}^{t} (i_b(t) + i_c(t))\, dt \tag{3.57}$$

The equivalent inductive reactance of the L||Cp element as

$$X_{eq} = \frac{X_{C_P} X_L}{X_{C_P} - X_L}$$

Using Eq (3.52) in Eq (3.57) and applying the initial conditions for the phase B and phase C capacitors obtained by neglecting the voltage drop across the source reactors and using voltage division,

$$v_{C_b}(t) + v_{C_c}(t) = v_{C_b}(0) + v_{C_c}(0) \approx \frac{-V_m}{2}\frac{X_C}{X_C - X_{eq}} + \frac{-V_m}{2}\frac{X_C}{X_C - X_{eq}} \tag{3.58}$$

$$v_{C_b}(t) + v_{C_c}(t) \approx -V_m \frac{X_C}{X_C - X_{eq}} \tag{3.59}$$

The voltage across the phase B and phase C parallel capacitors are given by,

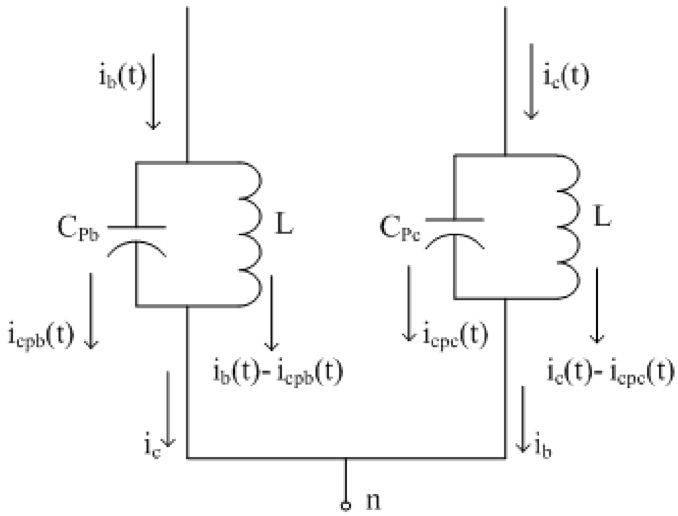

Figure 3.7: Circuit for Estimation of $v_{C_{Pb}}(t) + v_{C_{Pc}}(t)$

$$v_{C_{Pb}}(t) = L\frac{d\left(i_b(t) - i_{cpb}(t)\right)}{dt} \tag{3.60}$$

$$v_{C_{Pc}}(t) = L\frac{d\left(i_c(t) - i_{cpc}(t)\right)}{dt} \tag{3.61}$$

Adding Eq (3.60) and Eq (3.61),

$$v_{C_{Pb}}(t) + v_{C_{Pc}}(t) = L\frac{d\left(i_b(t) + i_c(t) - i_{cpb}(t) - i_{cpc}(t)\right)}{dt} \tag{3.62}$$

Using Eq (3.52) in Eq (3.62),

$$v_{C_{Pb}}(t) + v_{C_{Pc}}(t) = -L\frac{d\left(i_{cpb}(t) + i_{cpc}(t)\right)}{dt} \tag{3.63}$$

The currents flowing into the parallel capacitors are given as,

$$i_{cpb}(t) = C_P \frac{dv_{C_{Pb}}(t)}{dt} \tag{3.64}$$

$$i_{cpc}(t) = C_P \frac{dv_{C_{Pc}}(t)}{dt} \tag{3.65}$$

Adding Eq (3.64) and Eq (3.65),

$$i_{cpb}(t) + i_{cpc}(t) = C_P \frac{dv_{C_{Pb}}(t)}{dt} + C_P \frac{dv_{C_{Pc}}(t)}{dt} = C_P \frac{d\left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right)}{dt} \tag{3.66}$$

Substituting Eq (3.66) in Eq (3.63),

$$v_{C_{Pb}}(t) + v_{C_{Pc}}(t) = -LC_P \frac{d^2\left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right)}{dt^2} \tag{3.67}$$

Let us define $\omega_P = \frac{1}{\sqrt{LC_P}}$, and rewrite Eq (3.67).

$$\frac{d^2\left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right)}{dt^2} + {\omega_P}^2\left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right) = 0 \tag{3.68}$$

Taking the Laplace transform on both sides of Eq (3.68),

$$\begin{aligned} &s^2\left(V_{C_{Pb}}(s) + V_{C_{Pc}}(s)\right) - s\left(v_{C_{Pb}}(0) + v_{C_{Pc}}(0)\right) \\ &-\left(v'_{C_{Pb}}(0) + v'_{C_{Pc}}(0)\right) + {\omega_P}^2\left(V_{C_{Pb}}(s) + V_{C_{Pc}}(s)\right) = 0 \end{aligned} \tag{3.69}$$

Using current division,

$$i_{cpb}(0) = i_b(0) \frac{X_L}{X_L - X_{C_P}} \tag{3.70}$$

$$i_{cpc}(0) = i_c(0) \frac{X_L}{X_L - X_{C_P}} \tag{3.71}$$

Adding Eq (3.70) and Eq (3.71),

$$i_{cpb}(0) + i_{cpc}(0) = (i_b(0) + i_c(0)) \frac{X_L}{X_L - X_{C_P}} \tag{3.72}$$

Using Eq (3.52) in Eq (3.72),

$$i_{cpb}(0) + i_{cpc}(0) = 0 \tag{3.73}$$

Hence,

$$v'_{C_{Pb}}(0) + v'_{C_{Pc}}(0) = \frac{i_{cpb}(0) + i_{cpc}(0)}{C_P} = 0 \tag{3.74}$$

Substituting Eq (3.74) in Eq (3.69),

$$\left(s^2 + {\omega_P}^2\right)(V_{C_{Pb}}(s) + V_{C_{Pc}}(s)) = s\,(v_{C_{Pb}}(0) + v_{C_{Pc}}(0)) \tag{3.75}$$

$$(V_{C_{Pb}}(s) + V_{C_{Pc}}(s)) = (v_{C_{Pb}}(0) + v_{C_{Pc}}(0)) \frac{s}{(s^2 + {\omega_P}^2)} \tag{3.76}$$

Taking the inverse Laplace transform on both sides of Eq (3.76),

$$(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) = (v_{C_{Pb}}(0) + v_{C_{Pc}}(0))\, cos\omega_P t \quad \forall t \geq 0 \tag{3.77}$$

Applying the initial conditions for the phase B and phase C parallel capacitors obtained by neglecting the voltage drop across the source reactors and using voltage division,

$$v_{C_{Pb}}(0) = v_{C_{Pc}}(0) \approx \frac{-V_m}{2} \frac{-X_{eq}}{X_C - X_{eq}} \tag{3.78}$$

$$v_{C_{Pb}}(0) + v_{C_{Pc}}(0) \approx 2\frac{-V_m}{2} \frac{-X_{eq}}{X_C - X_{eq}} = V_m \frac{X_{eq}}{X_C - X_{eq}} \tag{3.79}$$

Substituting Eq (3.79) in Eq (3.77),

$$(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) = V_m \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t \tag{3.80}$$

Rewriting Eq (3.54) and rearranging it to estimate the neutral voltage,

$$-v_a(t) = (v_{C_b}(t) + v_{C_c}(t)) + (v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) + 2v_n(t) \tag{3.81}$$

$$v_n(t) = -\frac{1}{2}\left((v_{C_b}(t) + v_{C_c}(t)) + (v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) + v_a(t)\right) \tag{3.82}$$

Substituting Eq (3.80) and Eq (3.59) in Eq (3.82),

$$v_n(t) = -\frac{1}{2}\left(-V_m \frac{X_C}{X_C - X_{eq}} + V_m \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t + V_m cos\omega t\right) \tag{3.83}$$

$$v_n(t) = \frac{V_m}{2}\left(\frac{X_C}{X_C - X_{eq}} - \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t - cos\omega t\right) \quad \forall t \geq 0^+ \tag{3.84}$$

Recovery Voltage

The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at time $t = 0$ is given by Eq (1.11). This equation is applied to the phase A circuit.

$$v_{C_{Sa}}(t) = V_m cos\omega t - \left[V_m - v_{C_{Sa}}(0)\right] cos\omega_0 t \quad \forall t \geq 0 \tag{3.85}$$

Neglecting the voltage drop in the source reactance,

$$v_{C_{Sa}}(0) \approx V_m \tag{3.86}$$

$$v_{C_{Sa}}(t) = V_m cos\omega t \tag{3.87}$$

Using voltage division,

$$v_{C_a}(t) = v_{C_a}(0) \approx V_m \frac{X_C}{X_C - X_{eq}} \quad \forall t \geq 0 \tag{3.88}$$

The transient voltage that appears across the parallel capacitor after the breaker opening is given by Eq (1.24). This expression is used to estimate the phase A L||Cp element voltage.

$$v_{C_{Pa}}(t) = v_{C_{Pa}}(0) cos\omega_P t \quad \forall t \geq 0 \tag{3.89}$$

Using voltage division,

$$v_{C_{Pa}}(0) = \frac{-X_{eq}}{X_C - X_{eq}} V_m \tag{3.90}$$

$$v_{C_{Pa}}(t) = -V_m \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t \tag{3.91}$$

$$v_{TRV}(t) = v_{C_{Sa}}(t) - (v_{C_a}(t) + v_{C_{Pa}}(t) + v_n(t)) \tag{3.92}$$

$$\begin{aligned} v_{TRV}(t) =& V_m cos\omega t - V_m \frac{X_C}{X_C - X_{eq}} + V_m \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t \\ & - \frac{V_m}{2} \left(\frac{X_C}{X_C - X_{eq}} - \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t - cos\omega t \right) \end{aligned} \tag{3.93}$$

$$\begin{aligned} v_{TRV}(t) =& V_m cos\omega t - V_m \frac{X_C}{X_C - X_{eq}} + V_m \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t \\ & + \frac{V_m}{2} cos\omega t - \frac{V_m}{2} \frac{X_C}{X_C - X_{eq}} + \frac{V_m}{2} \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t \end{aligned} \tag{3.94}$$

$$v_{TRV}(t) = \frac{3}{2}V_m cos\omega t - \frac{3}{2}V_m \frac{X_C}{X_C - X_{eq}} + \frac{3}{2}V_m \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t \tag{3.95}$$

$$v_{TRV}(t) = \frac{3}{2}V_m \left(cos\omega t + \frac{X_{eq}}{X_C - X_{eq}} cos\omega_P t - \frac{X_C}{X_C - X_{eq}} \right) \quad \forall t \geq 0^+ \tag{3.96}$$

$$v_{TRV}(0^+) = \frac{3}{2}V_m \left(1 + \frac{X_{eq}}{X_C - X_{eq}} - \frac{X_C}{X_C - X_{eq}} \right) = 0 \tag{3.97}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{3.98}$$

$$v_{TRV}(0^-) = v_{TRV}(0^+) = 0 \tag{3.99}$$

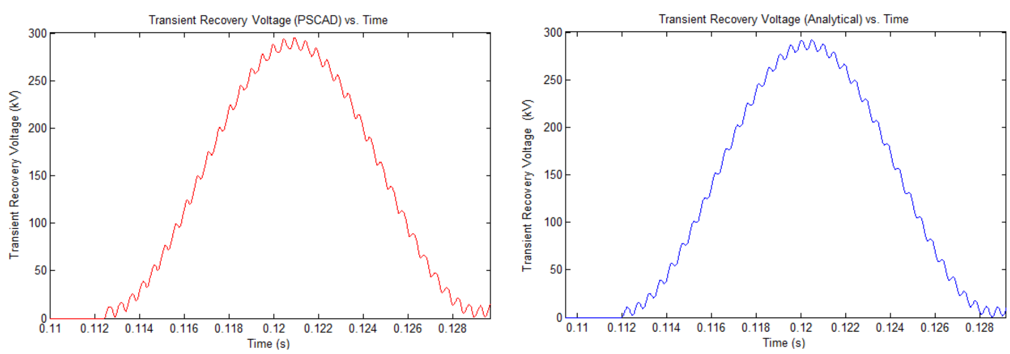

Figure 3.8: PSCAD and MATLAB Validation, De-energization of 3-Phase Capacitor Bank with Inrush Reactors - CB-L||Cp-C Configuration

Validation

$$L_s = 5 \ \text{mH}$$
$$C_s = 0.29 \ \mu\text{F}$$
$$L = 30 \ \text{mH}$$
$$C_P = 0.2005 \ \mu\text{F}$$
$$C = 8.4239 \ \mu\text{F}$$
$$V_m = 93.897 \ \text{kV}$$
$$\omega = 376.99 \ \text{rad/sec}$$
$$\omega_P = \frac{1}{\sqrt{LC_P}} = 12893.8 \ \text{rad/sec}$$

$$t_{open} = 0.112 \ \text{sec}$$
$$\forall t \geq t_{open}$$
$$v_{TRV}(t) = -\frac{3}{2}V_m\left(cos\omega(t-t_{open}) + \frac{X_{eq}}{X_C - X_{eq}}cos\omega_P(t-t_{open}) - \frac{X_C}{X_C - X_{eq}}\right)$$
$$v_{TRV}(t_{open}) = -\frac{3}{2}V_m\left(1 + \frac{X_{eq}}{X_C - X_{eq}} - \frac{X_C}{X_C - X_{eq}}\right) = 0$$

The significant component of the recovery voltage is the power frequency but a small transient frequency component is also observed. The small transient component appears due to the oscillation in the L||Cp element. There is no step jump in the recovery voltage at the instant the breaker opens. The parallel capacitor across the inrush reactor prevents a jump in the reactor voltage at the instant the breaker opens. The recovery voltage is 1.5 times greater than that of the corresponding single-phase capacitor de-energization.

3.2 TRV Initiated during Clearing a Fault (Opening of the First Pole of the Breaker)

The circuit breaker pole of phase A opens at the current zero to clear the three-phase fault to neutral which has occurred on the load side of the breaker. The current drawn with the occurrence of the fault can be inductive or capacitive depending on both the configuration of the capacitor bank with inrush reactors and the relative location of the fault on the load side. The phase A source voltage is assumed to be $v_a(t) = V_m cos\omega t$. Hence when the breaker opens at $t = 0$, the phase A instantaneous voltage is at the positive maximum V_m while the current flowing through it is zero. At this instant, the source voltage of phase B and C are $-\frac{V_m}{2}$.

After the opening of the first pole, there is asymmetry in the circuit and the neutral is no more at the ground potential. In fact, a transient is initiated for the neutral point voltage. Hence calculations involve estimating the transient neutral point or the fault point voltage with respect to ground followed by estimation of the transient recovery voltage.

3.2.1 3-Phase Fault to Neutral at Load Side Terminals of CB - All Capacitor Configurations

Before the incidence of the fault, by symmetry

$$v_n = v_g \Rightarrow v_{C_n} = 0 \tag{3.100}$$

Therefore,

$$Q_n = 0 \tag{3.101}$$

The neutral to ground capacitance C_n, does not have any stored charge.

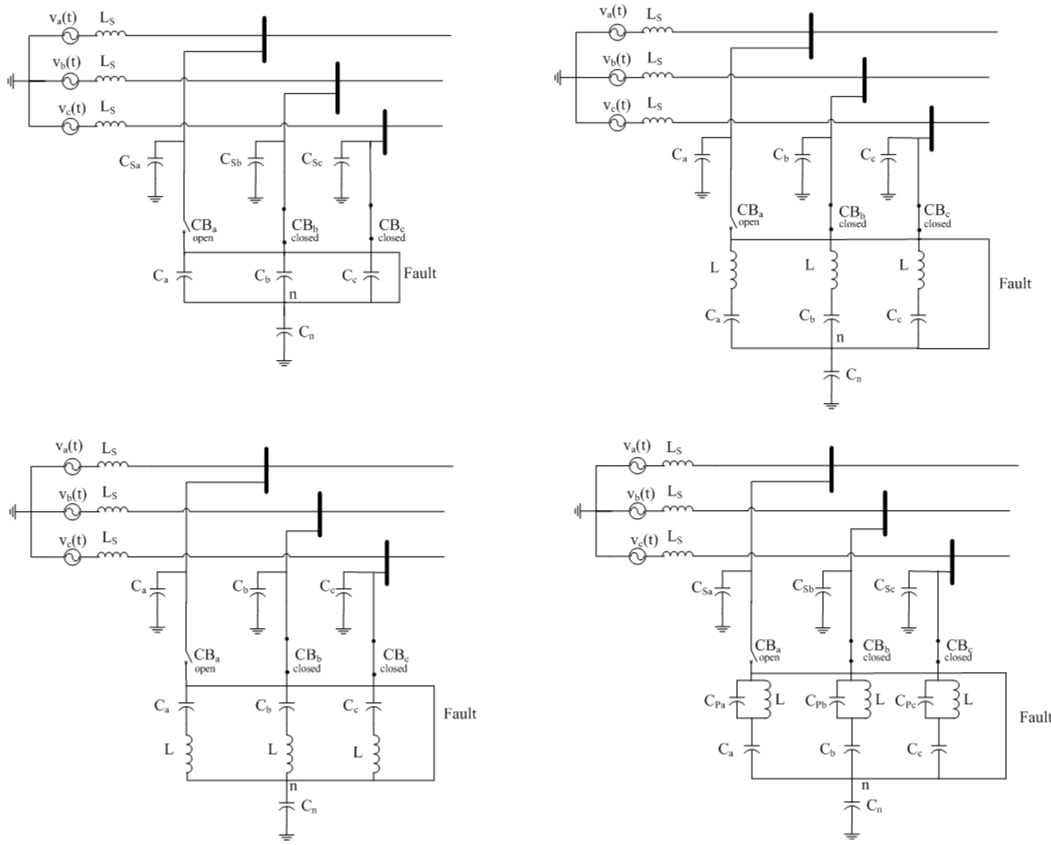

Figure 3.9: 3-Phase Fault to Neutral at Load Side Terminals of CB - All Capacitor Configurations

At any instant before the incidence of the fault, a balanced three-phase voltage appears across the three-phase capacitor bank,

$$v_{C_a} + v_{C_b} + v_{C_c} = 0 \tag{3.102}$$

$$Q_a + Q_b + Q_c = \frac{v_{C_a}}{C_a} + \frac{v_{C_b}}{C_b} + \frac{v_{C_c}}{C_c} \tag{3.103}$$

$$Q_a + Q_b + Q_c = \frac{v_{C_a} + v_{C_b} + v_{C_c}}{C} = 0 \tag{3.104}$$

After the fault occurs, the charges in the bank redistribute. The redistribution occurs within the bank as the stray neutral to ground capacitance is small compared to the bank capacitance.

$$v_{C_a} = v_{C_b} = v_{C_c} = 0 \tag{3.105}$$

$$Q_a = Q_b = Q_c = 0 \tag{3.106}$$

From charge conservation,

$$Q_n = 0 \tag{3.107}$$

Since there is no charge in the neutral capacitance C_n, we can conclude

$$v_{C_n} = 0 \Rightarrow v_n = v_g = 0 \tag{3.108}$$

Hence the neutral is still at the ground potential. At this instant the fault has occurred but the breaker has not opened. But when the first pole opens, say phase A, the circuit is asymmetric and the neutral point and the fault point voltage is hence not zero with respect to the ground.

The opening of the breaker pole initiates a transient due to presence of the stray capacitance C_S from the breaker terminal to the ground. The voltage across the stray capacitance for phase A is calculated just as in the analysis of a single-phase circuit, but the TRV is different as the neutral point is floating and the fault point voltage is not at ground potential and is nonzero.

The TRV is the difference between the phase A stray capacitance voltage and the fault point voltage and hence is different from the grounded fault case due to the floating neutral point. The asymmetry in the circuit also results in a transient involving the stray capacitances of the other two phases when the breaker pole on phase A opens.

For the analysis it is assumed that the neutral to ground capacitance is very small and hence the floating neutral voltage changes instantly without the effect of capacitive inertia. Hence the neutral to ground capacitance is merely used here to represent the voltage difference between ground and neutral, and this potential is the voltage of the fault point with respect to the ground. Its effect is neglected compared to the stray capacitances from the breaker terminals to ground.

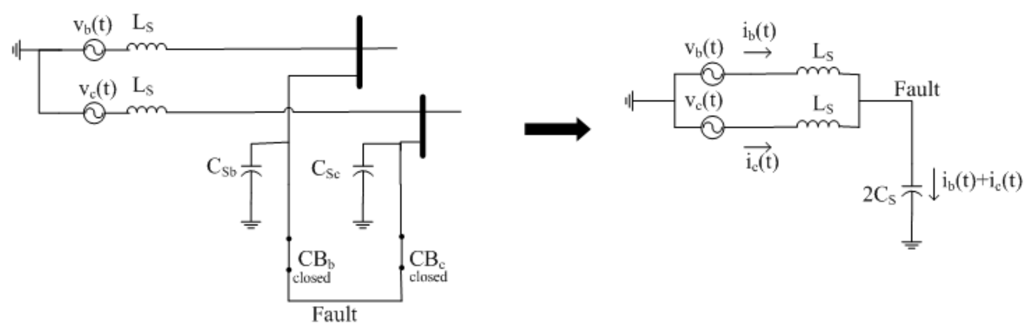

Figure 3.10: Equivalent circuit for Neutral Point Voltage Estimation, 3-Phase Fault to Neutral at Load Side Terminals of CB

Fault or Neutral Point Voltage Estimation

Referring to the equivalent circuit in Fig. 3.10 and applying KVL to the phase B and phase C circuits,

$$v_b(t) = L_S \frac{di_b(t)}{dt} + v_{2C_S}(t) \tag{3.109}$$

$$v_c(t) = L_S \frac{di_c(t)}{dt} + v_{2C_S}(t) \tag{3.110}$$

The fault or neutral point voltage is the voltage across the capacitance $2C_S$ as shown in the equivalent circuit.

$$v_{2C_S}(t) = v_{Fault}(t) = v_n(t) \tag{3.111}$$

Adding Eq (3.109) and Eq (3.110),

$$v_b(t) + v_c(t) = L_S \left(\frac{di_b(t)}{dt} + \frac{di_c(t)}{dt} \right) + 2v_{Fault}(t) \tag{3.112}$$

$$v_b(t) + v_c(t) = L_S \left(\frac{d(i_b(t) + i_c(t))}{dt} \right) + 2v_{Fault}(t) \tag{3.113}$$

The current flowing through the equivalent capacitor is given as,

$$i_b(t) + i_c(t) = 2C_S \frac{dv_{Fault}(t)}{dt} \tag{3.114}$$

For a balanced three-phase supply, the sum of the three phase voltages are zero at any instant of time and hence,

$$v_b(t) + v_c(t) = -v_a(t) \tag{3.115}$$

Substituting Eq (3.114) and Eq (3.115) in Eq (3.113),

$$-v_a(t) = 2L_S C_S \frac{d^2 v_{Fault}(t)}{dt^2} + 2v_{Fault}(t) \tag{3.116}$$

Let us define $\omega_0 = \frac{1}{\sqrt{L_S C_S}}$, and rewrite Eq (3.116).

$$\frac{d^2 v_{Fault}(t)}{dt^2} + {\omega_0}^2 v_{Fault}(t) = -\frac{{\omega_0}^2 v_a(t)}{2} \tag{3.117}$$

$$\frac{d^2 v_{Fault}(t)}{dt^2} + {\omega_0}^2 v_{Fault}(t) = -\frac{{\omega_0}^2 V_m cos(\omega t)}{2} \tag{3.118}$$

Taking the Laplace transform on both sides of Eq (3.118),

$$s^2 V_{Fault}(s) - s v_{Fault}(0) - {v_{Fault}}'(0) + {\omega_0}^2 V_{Fault}(s) = -\frac{{\omega_0}^2 V_m}{2} \frac{s}{s^2 + \omega^2} \tag{3.119}$$

The breaker pole of phase A opens at $t = 0$ and hence phase A line current is zero. The sum of all the three balanced line currents is zero at $t = 0$ and hence,

$${v_{Fault}}'(0) = \frac{i_b(0) + i_c(0)}{2C_S} = -\frac{i_a(0)}{2C_S} = 0 \tag{3.120}$$

The fault is a bolted fault to neutral. The fault or neutral point voltage is seen to be the voltage across the capacitor from the equivalent circuit. Since the voltage across the capacitor cannot change instantaneously we have

$$v_{Fault}(0) = v_n(0) = 0 \tag{3.121}$$

Substituting Eq (3.120) and Eq (3.121) in Eq (3.119),

$$\left(s^2 + {\omega_0}^2\right) V_{Fault}(s) = -\frac{{\omega_0}^2 V_m}{2} \frac{s}{s^2 + \omega^2} \tag{3.122}$$

$$V_{Fault}(s) = -\frac{{\omega_0}^2 V_m}{2} \frac{s}{\left(s^2 + \omega^2\right)\left(s^2 + {\omega_0}^2\right)} \tag{3.123}$$

Taking the inverse Laplace transform on both sides of Eq (3.123),

$$v_{Fault}(t) \approx -\frac{V_m}{2}\left[cos\omega t - cos\omega_0 t\right] \approx -\frac{V_m}{2}\left[1 - cos\omega_0 t\right] \quad \forall t \geq 0^+ \tag{3.124}$$

Estimation of TRV

The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at time $t = 0$ is given by Eq (1.11). This equation is applied to the phase A circuit.

$$v_{C_{Sa}}(t) = V_m cos\omega t - [V_m - v_{C_{Sa}}(0)]\, cos\omega_0 t \quad \forall t \geq 0 \tag{3.125}$$

The fault is a bolted fault to neutral and the neutral potential is zero at $t = 0$ and hence,

$$v_{C_{Sa}}(0) = 0 \tag{3.126}$$

$$v_{C_{Sa}}(t) = V_m cos\omega t - V_m cos\omega_0 t \approx V_m \left(1 - cos\omega_0 t\right) \tag{3.127}$$

$$v_{TRV}(t) = v_{C_{Sa}}(t) - v_{Fault}(t) \tag{3.128}$$

$$v_{TRV}(t) = V_m \left(1 - cos\omega_0 t\right) - \left(-\frac{V_m}{2}\left(1 - cos\omega_0 t\right)\right) \tag{3.129}$$

$$v_{TRV}(t) = \frac{3}{2} V_m \left(1 - cos\omega_0 t\right) \quad \forall t \geq 0^+ \tag{3.130}$$

$$v_{TRV}(0^+) = 0 \tag{3.131}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \quad (3.132)$$

$$v_{TRV}(0^-) = v_{TRV}(0^+) = 0 \quad (3.133)$$

The TRV for a three-phase ungrounded fault to neutral occurring in a three-phase ungrounded capacitor bank with a floating neutral is 1.5 times the TRV for a three-phase grounded fault occurring in a grounded capacitor bank configuration.

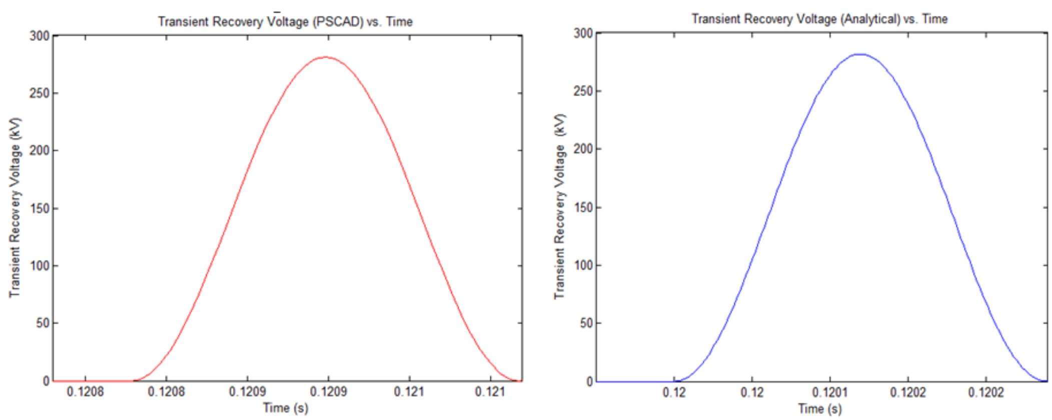

Figure 3.11: PSCAD and MATLAB Validation, 3-Phase Fault to Neutral at Load Side Terminals of CB - All Configurations

Validation

$$L_s = 5 \text{ mH}$$

$$C_s = 0.29 \ \mu\text{F}$$

$$C = 8.4239 \ \mu\text{F}$$

$$V_m = 93.897 \text{ kV}$$

$$\omega = 376.99 \text{ rad/sec}$$

$$\omega_0 = \frac{1}{\sqrt{L_S C_S}} = 26261 \text{ rad/sec}$$

$$t_{open} = 0.12 \text{ sec}$$

$$\forall t \geq t_{open}$$

$$v_{TRV}(t) \approx \frac{3}{2} V_m \left(1 - cos\omega_0 \left(t - t_{open}\right)\right)$$

$$v_{TRV}(t_{open}) = 0$$

$$v_{TRV,Peak} = 3V_m = 281.691 \text{ kV}$$

The recovery voltage constitutes a high frequency component and a power frequency component, but the power frequency term is assumed to be a constant for the small transient time span of interest. There is no step jump in the TRV at the instant the breaker pole of phase A opens. The stray capacitance at the breaker pole of phase A terminal has a zero initial condition at time t_{open}. The TRV is 1.5 times greater than that of the corresponding single-phase fault clearing case.

3.2.2 CB-L-C Configuration - 3-Phase Fault to Neutral between L and C Terminals

The equivalent circuit for estimating the neutral point or fault point voltage after the opening of the first breaker pole is shown in Figure 3.13. It is seen from the equivalent circuit that the source currents have two possible paths, through the stray capacitors and the inrush reactors L .

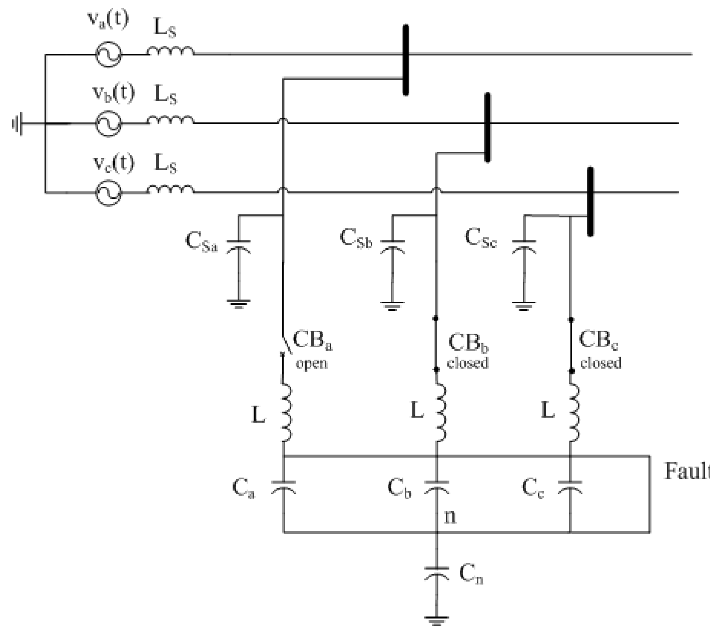

Figure 3.12: CB-L-C Configuration, 3-Phase Fault to Neutral between L and C Terminals

Estimation of Neutral Point or Fault Point Voltage

Referring to the equivalent circuit in Fig. 3.13 and applying KVL to the phase B and phase C circuits,

$$v_b(t) = L_S \frac{di_{b1}(t)}{dt} + L \frac{di_{b2}(t)}{dt} + v_{Fault}(t) \tag{3.134}$$

$$v_c(t) = L_S \frac{di_{c1}(t)}{dt} + L \frac{di_{c2}(t)}{dt} + v_{Fault}(t) \tag{3.135}$$

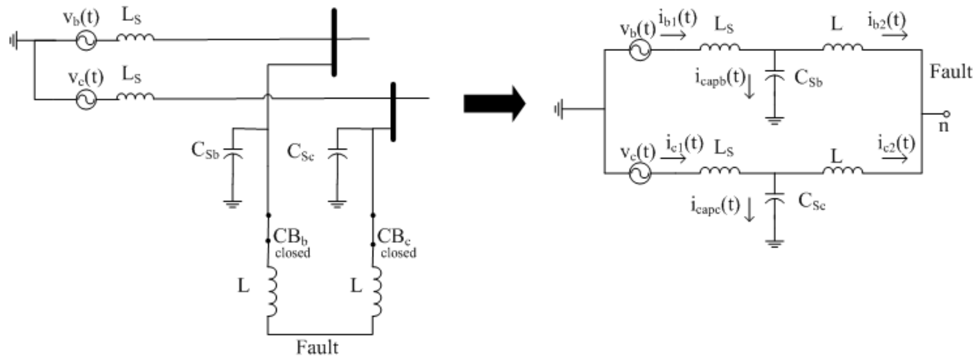

Figure 3.13: Equivalent Circuit for Neutral Voltage Estimation, CB-L-C Configuration, 3-Phase Fault to Neutral between L and C Terminals

Adding Eq (3.134) and Eq (3.135),

$$v_b(t) + v_c(t) = L_S\left(\frac{di_{b1}(t)}{dt} + \frac{di_{c1}(t)}{dt}\right) + L\left(\frac{di_{b2}(t)}{dt} + \frac{di_{c2}(t)}{dt}\right) + 2v_{Fault}(t) \tag{3.136}$$

$$v_b(t) + v_c(t) = L_S\left(\frac{d(i_{b1}(t) + i_{c1}(t))}{dt}\right) + L\left(\frac{d(i_{b2}(t) + i_{c2}(t))}{dt}\right) + 2v_{Fault}(t) \tag{3.137}$$

From KCL applied at the neutral n,

$$i_{b2}(t) + i_{c2}(t) = 0 \quad \forall t \geq 0 \tag{3.138}$$

For a balanced three-phase supply, the sum of the three phase voltages are zero at any instant of time and hence,

$$v_b(t) + v_c(t) = -v_a(t) \tag{3.139}$$

Substituting Eq (3.138) and Eq (3.139) in Eq (3.137),

$$-v_a(t) = L_S \left(\frac{d(i_{b1}(t) + di_{c1}(t))}{dt} \right) + 2v_{Fault}(t) \tag{3.140}$$

Referring to the equivalent circuit in Fig. 3.13, the sum of the souce currents $i_{b1}(t)$ and $i_{c1}(t)$ can be written as,

$$i_{b1}(t) + i_{c1}(t) = (i_{b2}(t) + i_{capb}(t)) + (i_{c2}(t) + i_{capc}(t)) \tag{3.141}$$

$$i_{b1}(t) + i_{c1}(t) = (i_{b2}(t) + i_{c2}(t)) + (i_{capb}(t) + i_{capc}(t)) \tag{3.142}$$

Using Eq (3.138) in Eq (3.142),

$$i_{b1}(t) + i_{c1}(t) = (i_{capb}(t) + i_{capc}(t)) \tag{3.143}$$

Substituting Eq (3.143) in Eq (3.140),

$$-v_a(t) = L_S \left(\frac{d(i_{capb}(t) + di_{capc}(t))}{dt} \right) + 2v_{Fault}(t) \tag{3.144}$$

The currents flowing through the phase B and phase C stray capacitors are given as,

$$i_{capb}(t) = C_S \frac{dv_{C_{Sb}}(t)}{dt} \tag{3.145}$$

$$i_{capc}(t) = C_S \frac{dv_{C_{Sc}}(t)}{dt} \tag{3.146}$$

Using Eq (3.145) and Eq (3.146) in Eq (3.144),

$$-v_a(t) = L_S C_S \frac{d^2 v_{C_{Sb}}}{dt^2} + L_S C_S \frac{d^2 v_{C_{Sc}}}{dt^2} + 2v_{Fault}(t) \tag{3.147}$$

Let us define $\omega_0 = \frac{1}{\sqrt{L_S C_S}}$, and rewrite Eq (3.147).

$$-{\omega_0}^2 v_a(t) = \frac{d^2 v_{C_{Sb}}}{dt^2} + \frac{d^2 v_{C_{Sc}}}{dt^2} + 2{\omega_0}^2 v_{Fault}(t) \tag{3.148}$$

$$-{\omega_0}^2 V_m cos(\omega t) = \frac{d^2 v_{C_{Sb}}}{dt^2} + \frac{d^2 v_{C_{Sc}}}{dt^2} + 2{\omega_0}^2 v_{Fault}(t) \tag{3.149}$$

Taking the Laplace transform on both sides of Eq (3.149),

$$\begin{aligned} -{\omega_0}^2 V_m \frac{s}{s^2+\omega^2} = &\left(s^2 V_{C_{Sb}}(s) - s v_{C_{Sb}}(0) - {v_{C_{Sb}}}'(0)\right) \\ &+ \left(s^2 V_{C_{Sc}}(s) - s v_{C_{Sc}}(0) - {v_{C_{Sc}}}'(0)\right) + 2{\omega_0}^2 V_{Fault}(s) \end{aligned} \tag{3.150}$$

$$\begin{aligned} -{\omega_0}^2 V_m \frac{s}{s^2+\omega^2} =& s^2 \left(V_{C_{Sb}}(s) + V_{C_{Sc}}(s)\right) - s\left(v_{C_{Sb}}(0) + v_{C_{Sc}}(0)\right) \\ &- \left({v_{C_{Sb}}}'(0) + {v_{C_{Sc}}}'(0)\right) + 2{\omega_0}^2 V_{Fault}(s) \end{aligned} \tag{3.151}$$

Expressions within each parenthesis of Eq (3.151) are evaluated.

From KVL, the voltages across the phase B and phase C stray capacitors are given as,

$$v_{C_{Sb}}(t) = L\frac{di_{b2}(t)}{dt} + v_{Fault}(t) \tag{3.152}$$

$$v_{C_{Sc}}(t) = L\frac{di_{c2}(t)}{dt} + v_{Fault}(t) \tag{3.153}$$

Adding Eq (3.152) and Eq (3.153),

$$v_{C_{Sb}}(t) + v_{C_{Sc}}(t) = L\frac{d\left(i_{b2}(t) + i_{c2}(t)\right)}{dt} + 2v_{Fault}(t) \tag{3.154}$$

Using Eq (3.138) in Eq (3.154),

$$v_{C_{Sb}}(t) + v_{C_{Sc}}(t) = 2v_{Fault}(t) \tag{3.155}$$

Taking the Laplace transform on both sides of Eq (3.155),

$$V_{C_{Sb}}(s) + V_{C_{Sc}}(s) = 2V_{Fault}(s) \tag{3.156}$$

The initial conditions for the phase B and phase C stray capacitors are obtained using voltage division.

$$v_{C_{Sb}}(0) = v_{C_{Sc}}(0) = -\frac{V_m}{2}\frac{L}{L + L_S} \tag{3.157}$$

$$v_{C_{Sb}}(0) + v_{C_{Sc}}(0) = -V_m\frac{L}{L + L_S} \tag{3.158}$$

Using the results from Eq (3.143) at time $t = 0$ on the right hand side of the following equation,

$$\begin{aligned} {v_{C_{Sb}}}'(0) + {v_{C_{Sc}}}'(0) &= \frac{i_{capb}(0)}{C_S} + \frac{i_{capc}(0)}{C_S} \\ &= \frac{\left(i_{capb}(0) + i_{capc}(0)\right)}{C_S} \\ &= \frac{\left(i_{b1}(0) + i_{c1}(0)\right)}{C_S} \end{aligned} \tag{3.159}$$

The breaker pole of phase A opens at $t = 0$ and hence phase A line current is zero. The sum of all the three balanced line currents is zero at $t = 0$ and hence,

$$v_{C_{Sb}}{}'(0) + v_{C_{Sc}}{}'(0) = \frac{(i_{b1}(0) + i_{c1}(0))}{C_S} = \frac{-i_{a1}(0)}{C_S} = 0 \tag{3.160}$$

Substituting results from Eq (3.156), Eq (3.158) and Eq (3.160) in Eq (3.151)

$$-\omega_0{}^2 V_m \frac{s}{s^2+\omega^2} = s^2\left(2V_{Fault}(s)\right) - s\left(-V_m \frac{L}{L+L_S}\right) + 2\omega_0{}^2 V_{Fault}(s) \tag{3.161}$$

$$-\omega_0{}^2 V_m \frac{s}{s^2+\omega^2} - V_m \frac{L}{L+L_S} s = 2V_{Fault}(s)\left(s^2+\omega_0{}^2\right) \tag{3.162}$$

$$V_{Fault}(s) = -\frac{\omega_0{}^2 V_m}{2} \frac{s}{\left(s^2+\omega^2\right)\left(s^2+\omega_0{}^2\right)} - \frac{V_m}{2} \frac{L}{L+L_S} \frac{s}{s^2+\omega_0{}^2} \tag{3.163}$$

Taking the inverse Laplace transform on both sides of Eq (3.163),

$$v_{Fault}(t) = -\frac{V_m}{2}\left(cos\omega t - cos\omega_0 t\right) - \frac{V_m}{2}\frac{L}{L+L_S} cos\omega_0 t \tag{3.164}$$

$$v_{Fault}(t) \approx -\frac{V_m}{2} + \frac{V_m}{2}\left(1 - \frac{L}{L+L_S}\right) cos\omega_0 t \tag{3.165}$$

$$v_{Fault}(t) \approx -\frac{V_m}{2}\left(1 - \frac{L_S}{L+L_S} cos\omega_0 t\right) \quad \forall t \geq 0^+ \tag{3.166}$$

Estimation of TRV

The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at time $t = 0$ is given by Eq (1.11). This equation is applied to the phase A circuit.

$$v_{C_{Sa}}(t) = V_m cos\omega t - \left[V_m - v_{C_{Sa}}(0)\right] cos\omega_0 t \quad \forall t \geq 0 \tag{3.167}$$

The fault is a bolted fault to neutral and the neutral potential is zero just before the breaker pole opens at $t = 0^-$ and hence using voltage division,

$$v_{C_{Sa}}(0^-) = \frac{L}{L + L_S} V_m \tag{3.168}$$

The voltage across a capacitor cannot change instantaneously and hence

$$v_{C_{Sa}}(0^-) = v_{C_{Sa}}(0^+) = v_{C_{Sa}}(0) = \frac{L}{L + L_S} V_m \tag{3.169}$$

$$v_{C_{Sa}}(t) = V_m cos\omega t - \frac{L_S}{L + L_S} V_m cos\omega_0 t \tag{3.170}$$

$$v_{C_{Sa}}(t) \approx V_m \left(1 - \frac{L_S}{L + L_S} cos\omega_0 t\right) \tag{3.171}$$

$$v_{TRV}(t) = v_{C_{Sa}} - v_{Fault}(t) \tag{3.172}$$

$$v_{TRV}(t) = V_m \left(1 - \frac{L_S}{L + L_S} cos\omega_0 t\right) - \left(-\frac{V_m}{2} \left(1 - \frac{L_S}{L + L_S} cos\omega_0 t\right)\right) \tag{3.173}$$

$$v_{TRV}(t) = \frac{3}{2} V_m \left(1 - \frac{L_S}{L + L_S} cos\omega_0 t\right) \quad \forall t \geq 0^+ \tag{3.174}$$

$$v_{TRV}(0^+) = \frac{3}{2} V_m \frac{L}{L + L_S} \tag{3.175}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{3.176}$$

$$v_{TRV}(0^-) \neq v_{TRV}(0^+) \tag{3.177}$$

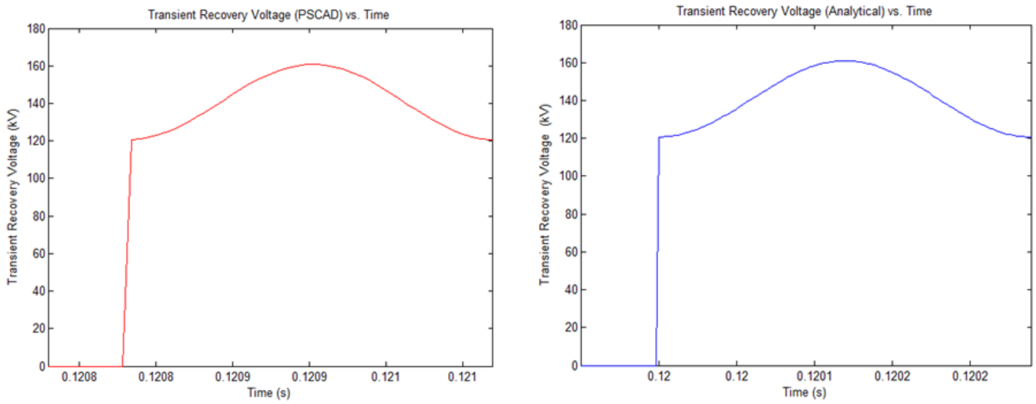

Figure 3.14: PSCAD and MATLAB Validation, CB-L-C Configuration, 3-Phase Fault to Neutral between L and C Terminals

Validation

$$L_s = 5 \text{ mH}$$

$$C_s = 0.29 \ \mu\text{F}$$

$$L = 30 \text{ mH}$$

$$C = 8.4239 \ \mu\text{F}$$

$$V_m = 93.897 \text{ kV}$$

$$\omega = 376.99 \text{ rad/sec}$$

$$\omega_0 = \frac{1}{\sqrt{L_S C_S}} = 26261 \text{ rad/sec}$$

$$
\begin{aligned}
t_{open} &= 0.12 \ \text{sec} \\
\forall t &\geq t_{open}^{+} \\
v_{TRV}(t) &\approx \frac{3}{2} V_m \left(1 - \frac{L_S}{L + L_S} cos\omega_0 \left(t - t_{open}\right)\right) \\
v_{TRV}(t_{open}^{+}) &= \frac{3}{2} V_m \left(\frac{L}{L + L_S}\right) = 120.725 \ \text{kV} \\
v_{TRV,Peak} &= \frac{3}{2} V_m \left(1 + \frac{L_S}{L + L_S}\right) = 160.966 \ \text{kV}
\end{aligned}
$$

The recovery voltage constitutes a high frequency component and a power frequency component, but the power frequency term is assumed to be a constant for the small transient time span of interest. There is a significant step jump in the TRV at the instant the breaker pole of phase A opens. The stray capacitance at the breaker pole of phase A terminal has a non-zero initial condition at time t_{open}. The breaker interrupts the current flowing through the inrush reactor when it opens. There is also a step jump in the neutral potential at the switching instant which causes the step jump in the TRV to be 1.5 times more as compared to the corresponding single-phase case. The recovery voltage is 1.5 times greater than that of the corresponding single-phase fault clearing case.

3.2.3 CB-C-L Configuration - 3-Phase Fault to Neutral between C and L Terminals

The inrush reactors are shorted by the fault. The breaker interrupts capacitive current when it opens. The circuit reduces to the same as the de-energization of a 3-phase capacitor bank without inrush reactors. Hence the previously derived results are directly used in this section.

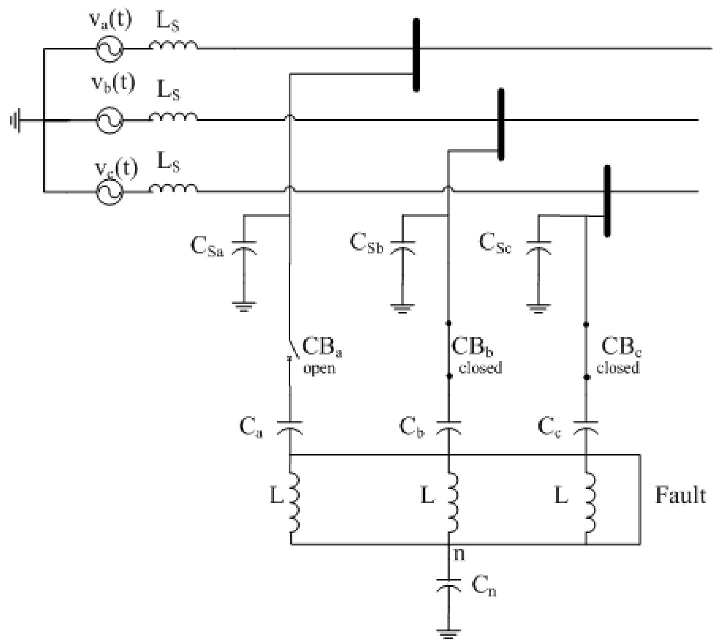

Figure 3.15: CB-C-L Configuration, 3-Phase Fault to Neutral between C and L Terminals

Estimation of TRV

The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at time $t = 0$ is given by Eq (1.11). This equation is applied to the phase A circuit.

$$v_{C_{Sa}}(t) = V_m cos\omega t - [V_m - v_{C_{Sa}}(0)]\, cos\omega_0 t \quad \forall t \geq 0 \tag{3.178}$$

Neglecting the voltage drop in the source reactance,

$$v_{C_a}(t) = v_{C_a}(0) = v_{C_{Sa}}(0) \approx V_m \quad \forall t \geq 0 \tag{3.179}$$

The neutral or fault point voltage is given by Eq (3.15) and is given below.

$$v_{Fault}(t) = v_n(t) = \frac{V_m}{2}\left(1 - cos\omega t\right) \tag{3.180}$$

$$v_{TRV}(t) = v_{C_{Sa}}(t) - \left(v_{C_a}(t) + v_{Fault}(t)\right) \tag{3.181}$$

$$v_{TRV}(t) = V_m cos\omega t - \left(V_m + \frac{V_m}{2}\left(1 - cos\omega t\right)\right) \tag{3.182}$$

$$v_{TRV}(t) = \frac{3}{2}V_m\left(cos\omega t - 1\right) \quad \forall t \geq 0^+ \tag{3.183}$$

$$v_{TRV}(0^+) = 0 \tag{3.184}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{3.185}$$

$$v_{TRV}(0^-) = v_{TRV}(0^+) = 0 \tag{3.186}$$

Validation

$$\begin{aligned} L_s &= 5 \ \text{mH} \\ C_s &= 0.29 \ \mu\text{F} \\ L &= 30 \ \text{mH} \\ C &= 8.4239 \ \mu\text{F} \\ V_m &= 93.897 \ \text{kV} \\ \omega &= 376.99 \ \text{rad/sec} \end{aligned}$$

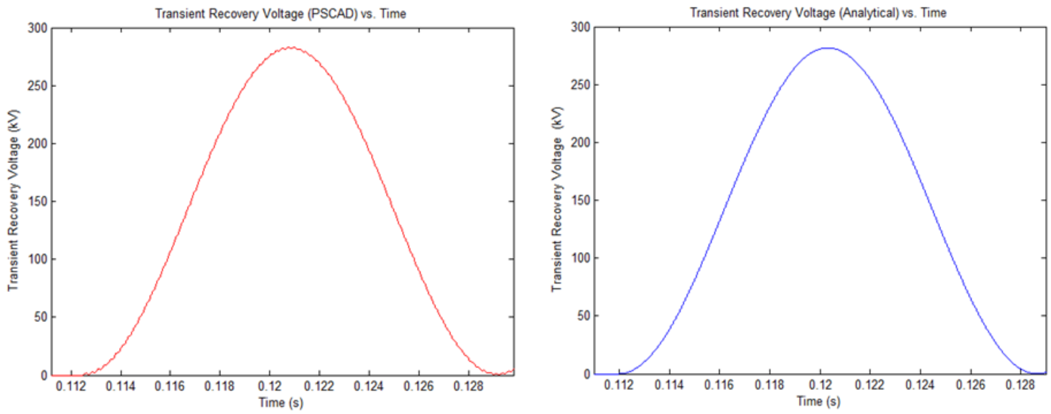

Figure 3.16: PSCAD and MATLAB Validation, CB-C-L Configuration, 3-Phase Fault to Neutral between C and L Terminals

$$t_{open} = 0.112 \text{ sec}$$

$$\forall t \geq t_{open}$$

$$v_{TRV}(t) = -\frac{3}{2}V_m \left(cos\omega \left(t - t_{open}\right) - 1\right)$$

$$v_{TRV}(t_{open}) = 0$$

$$v_{TRV,Peak} = 3V_m = 281.691 \text{ kV}$$

It is evident that the recovery voltage is of power frequency and has a negligible transient component. Hence it is better called as power frequency recovery voltage. There is no step jump in the recovery voltage at the instant the breaker pole of phase A opens. The recovery voltage is 1.5 times greater than that of the corresponding single-phase fault clearing case.

3.2.4 CB-L||Cp-C Configuration - 3-Phase Fault to Neutral between L||Cp and C Terminals

The equivalent circuit for estimating the neutral point or fault point voltage after the opening of the first breaker pole is shown in Figure 3.18. It is seen from the equivalent circuit that the source currents have two possible paths, through the stray capacitors and the L||Cp elements .

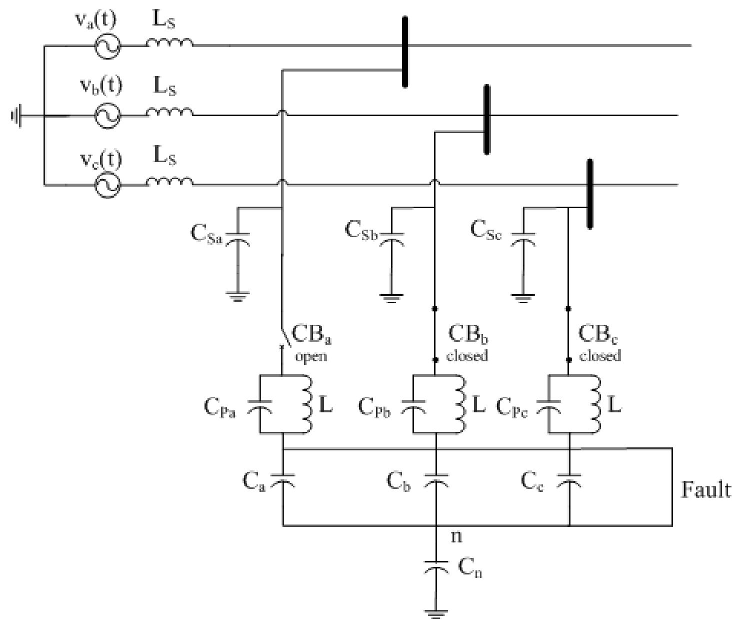

Figure 3.17: CB-L||Cp-C Configuration, 3-Phase Fault to Neutral at Terminals of L||Cp and C

Estimation of Neutral Point or Fault Point Voltage

Referring to the equivalent circuit in Fig. 3.18 and applying KVL to the phase B and phase C circuits,

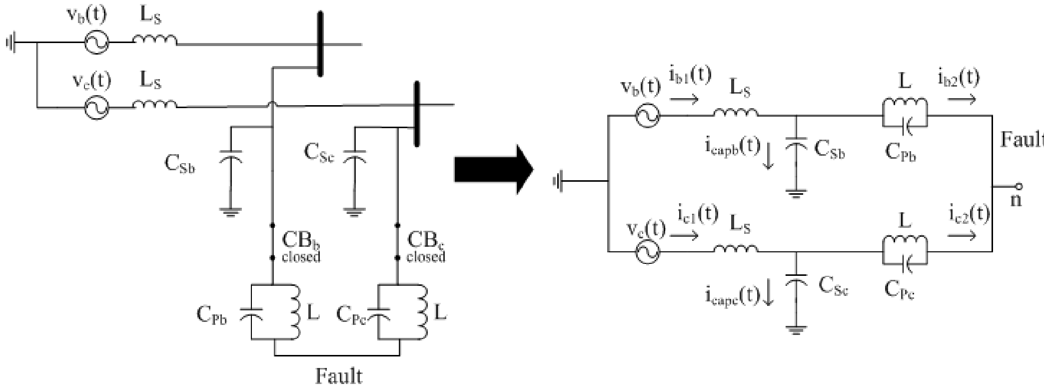

Figure 3.18: Equivalent Circuit for Neutral Voltage Estimation, CB-L||Cp-C Configuration, 3-Phase Fault to Neutral between L||Cp and C Terminals

$$v_b(t) = L_S \frac{di_{b1}(t)}{dt} + v_{C_{Pb}}(t) + v_{Fault}(t) \tag{3.187}$$

$$v_c(t) = L_S \frac{di_{c1}(t)}{dt} + v_{C_{Pc}}(t) + v_{Fault}(t) \tag{3.188}$$

Adding Eq (3.187) and Eq (3.188),

$$v_b(t)+v_c(t) = L_S \left(\frac{di_{b1}(t)}{dt} + \frac{di_{c1}(t)}{dt} \right) + (v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) + 2v_{Fault}(t) \tag{3.189}$$

$$v_b(t)+v_c(t) = L_S \left(\frac{d(i_{b1}(t) + i_{c1}(t))}{dt} \right) + (v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) + 2v_{Fault}(t) \tag{3.190}$$

From KCL applied at the neutral n,

$$i_{b2}(t) + i_{c2}(t) = 0 \quad \forall t \geq 0 \tag{3.191}$$

For a balanced three-phase supply, the sum of the three phase voltages are zero at any instant of time and hence,

$$v_b(t) + v_c(t) = -v_a(t) \tag{3.192}$$

Substituting Eq (3.191) and Eq (3.192) in Eq (3.190),

$$-v_a(t) = L_S\left(\frac{d(i_{b1}(t) + di_{c1}(t))}{dt}\right) + (v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) + 2v_{Fault}(t) \tag{3.193}$$

Referring to the equivalent circuit in Fig. 3.18, the sum of the souce currents $i_{b1}(t)$ and $i_{c1}(t)$ can be written as,

$$i_{b1}(t) + i_{c1}(t) = (i_{b2}(t) + i_{capb}(t)) + (i_{c2}(t) + i_{capc}(t)) \tag{3.194}$$

$$i_{b1}(t) + i_{c1}(t) = (i_{b2}(t) + i_{c2}(t)) + (i_{capb}(t) + i_{capc}(t)) \tag{3.195}$$

Using Eq (3.191) in Eq (3.195),

$$i_{b1}(t) + i_{c1}(t) = (i_{capb}(t) + i_{capc}(t)) \tag{3.196}$$

Substituting Eq (3.196) in Eq (3.193),

$$-v_a(t) = L_S\left(\frac{d(i_{capb}(t) + di_{capc}(t))}{dt}\right) + (v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) + 2v_{Fault}(t) \tag{3.197}$$

The currents flowing through the phase B and phase C stray capacitors are given as,

$$i_{capb}(t) = C_S\frac{dv_{C_{Sb}}(t)}{dt} \tag{3.198}$$

$$i_{capc}(t) = C_S\frac{dv_{C_{Sc}}(t)}{dt} \tag{3.199}$$

Using Eq (3.198) and Eq (3.199) in Eq (3.197),

$$-v_a(t) = L_S C_S \frac{d^2 v_{C_{Sb}}}{dt^2} + L_S C_S \frac{d^2 v_{C_{Sc}}}{dt^2} + (v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) + 2v_{Fault}(t) \quad (3.200)$$

Let us define $\omega_0 = \frac{1}{\sqrt{L_S C_S}}$, and rewrite Eq (3.200).

$$-{\omega_0}^2 v_a(t) = \frac{d^2 v_{C_{Sb}}}{dt^2} + \frac{d^2 v_{C_{Sc}}}{dt^2} + {\omega_0}^2 \left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right) + 2{\omega_0}^2 v_{Fault}(t) \quad (3.201)$$

$$-{\omega_0}^2 V_m cos(\omega t) = \frac{d^2 v_{C_{Sb}}}{dt^2} + \frac{d^2 v_{C_{Sc}}}{dt^2} + {\omega_0}^2 \left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right) + 2{\omega_0}^2 v_{Fault}(t) \quad (3.202)$$

Taking the Laplace transform on both sides of Eq (3.202),

$$\begin{aligned} -{\omega_0}^2 V_m \frac{s}{s^2+\omega^2} &= \left(s^2 V_{C_{Sb}}(s) - s v_{C_{Sb}}(0) - {v_{C_{Sb}}}'(0)\right) \\ &+ \left(s^2 V_{C_{Sc}}(s) - s v_{C_{Sc}}(0) - {v_{C_{Sc}}}'(0)\right) \\ &+ {\omega_0}^2 \left(V_{C_{Pb}}(s) + V_{C_{Pc}}(s)\right) + 2{\omega_0}^2 V_{Fault}(s) \end{aligned} \quad (3.203)$$

$$\begin{aligned} -{\omega_0}^2 V_m \frac{s}{s^2+\omega^2} =& s^2 \left(V_{C_{Sb}}(s) + V_{C_{Sc}}(s)\right) - s\left(v_{C_{Sb}}(0) + v_{C_{Sc}}(0)\right) \\ &- \left({v_{C_{Sb}}}'(0) + {v_{C_{Sc}}}'(0)\right) + {\omega_0}^2 \left(V_{C_{Pb}}(s) + V_{C_{Pc}}(s)\right) \\ &+ 2{\omega_0}^2 V_{Fault}(s) \end{aligned} \quad (3.204)$$

Expressions within each parenthesis of Eq (3.204) are evaluated.

From KVL, the voltages across the phase B and phase C stray capacitors are given as,

$$v_{C_{Sb}}(t) = v_{C_{Pb}}(t) + v_{Fault}(t) \tag{3.205}$$

$$v_{C_{Sc}}(t) = v_{C_{Pc}}(t) + v_{Fault}(t) \tag{3.206}$$

Adding Eq (3.205) and Eq (3.206),

$$v_{C_{Sb}}(t) + v_{C_{Sc}}(t) = (v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) + 2v_{Fault}(t) \tag{3.207}$$

Taking the Laplace transform on both sides of Eq (3.207),

$$V_{C_{Sb}}(s) + V_{C_{Sc}}(s) = (V_{C_{Pb}}(s) + V_{C_{Pc}}(s)) + 2V_{Fault}(s) \tag{3.208}$$

The initial conditions for the phase B and phase C stray capacitors are obtained using voltage division.

$$v_{C_{Sb}}(0) = v_{C_{Sc}}(0) = -\frac{V_m}{2}\frac{X_{eq}}{X_{eq} + X_{L_S}} \tag{3.209}$$

$$v_{C_{Sb}}(0) + v_{C_{Sc}}(0) = -V_m\frac{X_{eq}}{X_{eq} + X_{L_S}} \tag{3.210}$$

Using the results from Eq (3.196) at time $t = 0$ on the right hand side of the following equation,

$$\begin{aligned} {v_{C_{Sb}}}'(0) + {v_{C_{Sc}}}'(0) =& \frac{i_{capb}(0)}{C_S} + \frac{i_{capc}(0)}{C_S} \\ =& \frac{(i_{capb}(0) + i_{capc}(0))}{C_S} \\ =& \frac{(i_{b1}(0) + i_{c1}(0))}{C_S} \end{aligned} \tag{3.211}$$

The breaker pole of phase A opens at $t = 0$ and hence phase A line current is zero. The sum of all the three balanced line currents is zero at $t = 0$ and hence,

$$v_{C_{Sb}}{}'(0) + v_{C_{Sc}}{}'(0) = \frac{(i_{b1}(0) + i_{c1}(0))}{C_S} = \frac{-i_{a1}(0)}{C_S} = 0 \tag{3.212}$$

Substituting results from Eq (3.208), Eq (3.210) and Eq (3.212) in Eq (3.204),

$$\begin{aligned} -{\omega_0}^2 V_m \frac{s}{s^2+\omega^2} =& s^2\left((V_{C_{Pb}}(s) + V_{C_{Pc}}(s)) + 2V_{Fault}(s)\right) - s\left(-V_m \frac{X_{eq}}{X_{eq} + X_{L_S}}\right) \\ &+ {\omega_0}^2\left(V_{C_{Pb}}(s) + V_{C_{Pc}}(s)\right) + 2{\omega_0}^2 V_{Fault}(s) \end{aligned} \tag{3.213}$$

$$\begin{aligned} -{\omega_0}^2 V_m \frac{s}{s^2+\omega^2} =& \left(s^2 + {\omega_0}^2\right)\left(V_{C_{Pb}}(s) + V_{C_{Pc}}(s)\right) - s\left(-V_m \frac{X_{eq}}{X_{eq} + X_{L_S}}\right) \\ &+ \left(s^2 + {\omega_0}^2\right)\left(2V_{Fault}(s)\right) \end{aligned} \tag{3.214}$$

$$\begin{aligned} -{\omega_0}^2 V_m \frac{s}{s^2+\omega^2} - s\left(V_m \frac{X_{eq}}{X_{eq} + X_{L_S}}\right) =& \left(s^2 + {\omega_0}^2\right)\left(V_{C_{Pb}}(s) + V_{C_{Pc}}(s)\right) \\ &+ \left(s^2 + {\omega_0}^2\right)\left(2V_{Fault}(s)\right) \end{aligned} \tag{3.215}$$

$$V_{Fault}(s) = -\frac{{\omega_0}^2 V_m}{2} \frac{s}{(s^2+\omega^2)(s^2+{\omega_0}^2)} - \frac{V_m}{2}\frac{X_{eq}}{X_{eq} + X_{L_S}}\frac{s}{s^2+{\omega_0}^2} - \frac{V_{C_{Pb}}(s) + V_{C_{Pc}}(s)}{2} \tag{3.216}$$

Taking the inverse Laplace transform on both sides of Eq (3.216),

$$v_{Fault}(t) = -\frac{V_m}{2}\left(cos\omega t - cos\omega_0 t\right) - \frac{V_m}{2}\frac{X_{eq}}{X_{eq}+X_{L_S}}cos\omega_0 t - \frac{v_{C_{Pb}}(t)+v_{C_{Pc}}(t)}{2} \tag{3.217}$$

$$v_{Fault}(t) = -\frac{V_m}{2}cos\omega t + \frac{V_m}{2}\frac{X_{L_S}}{X_{eq}+X_{L_S}}cos\omega_0 t - \frac{v_{C_{Pb}}(t)+v_{C_{Pc}}(t)}{2} \tag{3.218}$$

The sum of the voltages $v_{C_{Pb}}(t)$ and $v_{C_{Pc}}(t)$ has to be estimated to obtain the expression for the fault point or neutral point voltage.

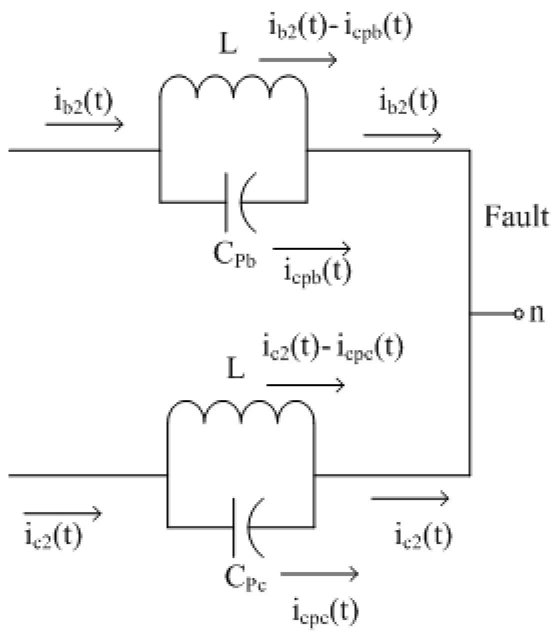

Figure 3.19: Circuit for Estimation of $v_{C_{Pb}}(t) + v_{C_{Pc}}(t)$

The voltage across the phase B and phase C parallel capacitors are given by,

$$v_{C_{Pb}}(t) = L\frac{d\left(i_{b2}(t) - i_{cpb}(t)\right)}{dt} \tag{3.219}$$

$$v_{C_{Pc}}(t) = L\frac{d\left(i_{c2}(t) - i_{cpc}(t)\right)}{dt} \tag{3.220}$$

Adding Eq (3.219) and Eq (3.220),

$$v_{C_{Pb}}(t) + v_{C_{Pc}}(t) = L\frac{d\left(i_{b2}(t) + i_{c2}(t) - i_{cpb}(t) - i_{cpc}(t)\right)}{dt} \tag{3.221}$$

Using Eq (3.191) in Eq (3.221),

$$v_{C_{Pb}}(t) + v_{C_{Pc}}(t) = -L\frac{d\left(i_{cpb}(t) + i_{cpc}(t)\right)}{dt} \tag{3.222}$$

The currents flowing into the parallel capacitors are given as,

$$i_{cpb}(t) = C_P\frac{dv_{C_{Pb}}(t)}{dt} \tag{3.223}$$

$$i_{cpc}(t) = C_P\frac{dv_{C_{Pc}}(t)}{dt} \tag{3.224}$$

Adding Eq (3.223) and Eq (3.224),

$$i_{cpb}(t)+i_{cpc}(t) = C_P\frac{dv_{C_{Pb}}(t)}{dt}+C_P\frac{dv_{C_{Pc}}(t)}{dt} = C_P\frac{d\left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right)}{dt} \tag{3.225}$$

Substituting Eq (3.225) in Eq (3.222),

$$v_{C_{Pb}}(t) + v_{C_{Pc}}(t) = -LC_P\frac{d^2\left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right)}{dt^2} \tag{3.226}$$

Let us define $\omega_P = \frac{1}{\sqrt{LC_P}}$, and rewrite Eq (3.226).

$$\frac{d^2\left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right)}{dt^2} + {\omega_P}^2\left(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)\right) = 0 \tag{3.227}$$

Taking the Laplace transform on both sides of Eq (3.227),

$$
\begin{aligned}
&s^2\left(V_{C_{Pb}}(s)+V_{C_{Pc}}(s)\right)-s\left(v_{C_{Pb}}(0)+v_{C_{Pc}}(0)\right)\\
&-\left(v'_{C_{Pb}}(0)+v'_{C_{Pc}}(0)\right)+{\omega_P}^2\left(V_{C_{Pb}}(s)+V_{C_{Pc}}(s)\right)=0
\end{aligned} \tag{3.228}
$$

Using current division,

$$
i_{cpb}(0)=i_{b2}(0)\frac{X_L}{X_L-X_{C_P}} \tag{3.229}
$$

$$
i_{cpc}(0)=i_{c2}(0)\frac{X_L}{X_L-X_{C_P}} \tag{3.230}
$$

Adding Eq (3.229) and Eq (3.230),

$$
i_{cpb}(0)+i_{cpc}(0)=\left(i_{b2}(0)+i_{c2}(0)\right)\frac{X_L}{X_L-X_{C_P}} \tag{3.231}
$$

Using Eq (3.191) in Eq (3.231),

$$
i_{cpb}(0)+i_{cpc}(0)=0 \tag{3.232}
$$

Hence,

$$
v'_{C_{Pb}}(0)+v'_{C_{Pc}}(0)=\frac{i_{cpb}(0)+i_{cpc}(0)}{C_P}=0 \tag{3.233}
$$

Substituting Eq (3.233) in Eq (3.228),

$$
\left(s^2+{\omega_P}^2\right)\left(V_{C_{Pb}}(s)+V_{C_{Pc}}(s)\right)=s\left(v_{C_{Pb}}(0)+v_{C_{Pc}}(0)\right) \tag{3.234}
$$

$$
\left(V_{C_{Pb}}(s)+V_{C_{Pc}}(s)\right)=\left(v_{C_{Pb}}(0)+v_{C_{Pc}}(0)\right)\frac{s}{(s^2+{\omega_P}^2)} \tag{3.235}
$$

Taking the inverse Laplace transform on both sides of Eq (3.235),

$$(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) = (v_{C_{Pb}}(0) + v_{C_{Pc}}(0))\, cos\omega_P t \tag{3.236}$$

Applying the initial conditions for the phase B and phase C parallel capacitors obtained by neglecting the voltage drop across the source reactors and using voltage division,

$$v_{C_{Pb}}(0) = v_{C_{Pc}}(0) = \frac{-V_m}{2}\frac{X_{eq}}{X_{eq} + X_{L_S}} \tag{3.237}$$

$$v_{C_{Pb}}(0) + v_{C_{Pc}}(0) = -V_m\frac{X_{eq}}{X_{eq} + X_{L_S}} \tag{3.238}$$

Substituting Eq (3.238) in Eq (3.236),

$$(v_{C_{Pb}}(t) + v_{C_{Pc}}(t)) = -V_m\frac{X_{eq}}{X_{eq} + X_{L_S}} cos\omega_P t \tag{3.239}$$

Substituting Eq (3.239) in Eq (3.218),

$$v_{Fault}(t) = -\frac{V_m}{2}cos\omega t + \frac{V_m}{2}\frac{X_{L_S}}{X_{eq} + X_{L_S}}cos\omega_0 t - \frac{1}{2}\left(-V_m\frac{X_{eq}}{X_{eq} + X_{L_S}}cos\omega_P t\right) \tag{3.240}$$

$$v_{Fault}(t) = -\frac{V_m}{2}cos\omega t + \frac{V_m}{2}\frac{X_{L_S}}{X_{eq} + X_{L_S}}cos\omega_0 t + \frac{V_m}{2}\frac{X_{eq}}{X_{eq} + X_{L_S}}cos\omega_P t \tag{3.241}$$

$$v_{Fault}(t) \approx -\frac{V_m}{2} + \frac{V_m}{2}\frac{X_{L_S}}{X_{eq} + X_{L_S}}cos\omega_0 t + \frac{V_m}{2}\frac{X_{eq}}{X_{eq} + X_{L_S}}cos\omega_P t \quad \forall t \geq 0^+ \tag{3.242}$$

Estimation of TRV

The transient voltage that appears across the stray capacitance C_S when the circuit breaker opens at time $t = 0$ is given by Eq (1.11). This equation is applied to the phase A circuit.

$$v_{C_{Sa}}(t) = V_m cos\omega t - [V_m - v_{C_{Sa}}(0)]\, cos\omega_0 t \quad \forall t \geq 0 \tag{3.243}$$

Using voltage division,

$$v_{C_{Sa}}(0) = \frac{X_{eq}}{X_{eq} + X_{L_S}} V_m \tag{3.244}$$

$$v_{C_{Sa}}(t) = V_m cos\omega t - \frac{X_{L_S}}{X_{eq} + X_{L_S}} V_m cos\omega_0 t \tag{3.245}$$

$$v_{C_{Sa}}(t) \approx V_m \left(1 - \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0 t\right) \tag{3.246}$$

The transient voltage that appears across the parallel capacitor after the breaker opening is given by Eq (1.24). This expression is used to estimate the phase A L||Cp element voltage.

$$v_{C_{Pa}}(t) = v_{C_{Pa}}(0) cos\omega_P t \quad \forall t \geq 0 \tag{3.247}$$

Using voltage division,

$$v_{C_{Pa}}(0) = \frac{X_{eq}}{X_{eq} + X_{L_S}} V_m \tag{3.248}$$

$$v_{C_{Pa}}(t) = V_m \frac{X_{eq}}{X_{eq} + X_{L_S}} cos\omega_P t \tag{3.249}$$

$$v_{TRV}(t) = v_{C_{Sa}} - \left(v_{C_{pa}} + v_{Fault}(t)\right) \tag{3.250}$$

$$\begin{aligned} v_{TRV}(t) =& V_m - V_m \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0 t - V_m \frac{X_{eq}}{X_{eq} + X_{L_S}} cos\omega_P t \\ &+ \frac{V_m}{2} - \frac{V_m}{2} \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0 t - \frac{V_m}{2} \frac{X_{eq}}{X_{eq} + X_{L_S}} cos\omega_P t \end{aligned} \tag{3.251}$$

$$v_{TRV}(t) = \frac{3}{2} V_m - \frac{3}{2} V_m \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0 t - \frac{3}{2} V_m \frac{X_{eq}}{X_{eq} + X_{L_S}} cos\omega_P t \tag{3.252}$$

$$v_{TRV}(t) = \frac{3}{2} V_m \left(1 - \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0 t - \frac{X_{eq}}{X_{eq} + X_{L_S}} cos\omega_P t\right) \quad \forall t \geq 0^+ \tag{3.253}$$

$$v_{TRV}(0^+) = \frac{3}{2} V_m \left(1 - \frac{X_{L_S}}{X_{eq} + X_{L_S}} - \frac{X_{eq}}{X_{eq} + X_{L_S}}\right) = 0 \tag{3.254}$$

Before the opening of the circuit breaker, there is no voltage drop across it.

$$v_{TRV}(0^-) = 0 \tag{3.255}$$

$$v_{TRV}(0^-) = v_{TRV}(0^+) = 0 \tag{3.256}$$

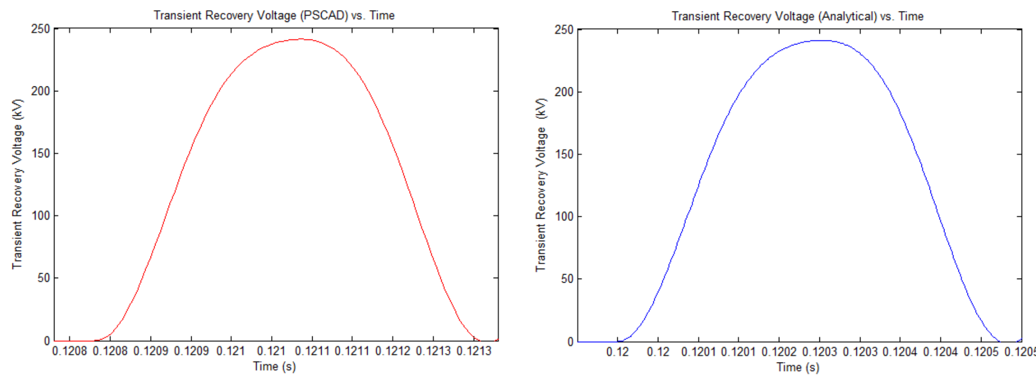

Figure 3.20: PSCAD and MATLAB Validation, CB-L||Cp-C Configuration, 3-Phase Fault to Neutral between L||Cp and C Terminals

Validation

$$L_s = 5 \text{ mH}$$

$$C_s = 0.29 \text{ } \mu\text{F}$$

$$L = 30 \text{ mH}$$

$$C_P = 0.2005 \text{ } \mu\text{F}$$

$$C = 8.4239 \text{ } \mu\text{F}$$

$$V_m = 93.897 \text{ kV}$$

$$\omega = 376.99 \text{ rad/sec}$$

$$\omega_0 = \frac{1}{\sqrt{L_S C_S}} = 26261 \text{ rad/sec}$$

$$\omega_P = \frac{1}{\sqrt{L C_P}} = 12893.8 \text{ rad/sec}$$

$$
\begin{aligned}
t_{open} &= 0.12 \text{ sec} \\
\forall t &\geq t_{open} \\
v_{TRV}(t) &= \frac{3}{2}V_m \left(1 - \frac{X_{L_S}}{X_{eq} + X_{L_S}} cos\omega_0(t - t_{open}) - \frac{X_{eq}}{X_{eq} + X_{L_S}} cos\omega_P(t - t_{open})\right) \\
v_{TRV}(t_{open}) &= \frac{3}{2}V_m \left(1 - \frac{X_{L_S}}{X_{eq} + X_{L_S}} - \frac{X_{eq}}{X_{eq} + L_S}\right) = 0
\end{aligned}
$$

The recovery voltage constitutes two high frequency components and a power frequency component, but the power frequency term is assumed to be a constant for the small transient time span of interest. The waveform constitutes both the high frequency components, but only the ω_P component is evident. ω_0 is about twice ω_P. The ω_0 component adds to the ω_P component at the rise and opposes at the peak. Hence the peak magnitude is less and the waveform is flat close to the peak. There is no step jump in the TRV at the instant the breaker opens. This is because of the parallel capacitor across the inrush reactor. The recovery voltage is 1.5 times greater than that of the corresponding single-phase fault clearing case.

Chapter 4

Conclusions

The installation of shunt capacitor banks in the power system results in high frequency and high magnitude inrush currents. The breaker connecting the shunt capacitor to the main line has to be rated to withstand the high frequency and magnitude of the inrush current. Installation of inrush current limiting reactors reduce the transient inrush currents during capacitor switching or energization. But the installation of inrush reactors leads to issues regarding the transient recovery voltage across the breaker, when it opens to clear the fault in the vicinity of the capcitor and inrush reactor terminals.

When the inrush reactor is installed between the breaker and the capacitor and a fault occurs between the inrush reactor and capacitor terminal (CB-L-C configuration, fault between the L and C terminals), the breaker has to interrupt the fault current flowing through the inrush reactor. This results in a step jump in the TRV seen across the breaker. The step jump is equivalent to a near infinite rate of rise of recovery voltage (RRRV). The dielectric strength across the breaker does not build fast enough to overcome the high RRRV and this results in breaker failure and the fault current is not interrupted.

One solution is to install the inrush reactors between the capacitor and the ground in single-phase systems and between the capacitors and the neutral in a three-phase system (CB-C-L configuration). This configuration prevents fault current from flowing through the reactor and hence no reactor current is interrupted when the breaker opens to clear the fault. No step jump in voltage appears across the breaker.

The other solution is to install a distribution size capacitor bank in parallel with the inrush reactor (CB-L||Cp-C configuration). The presence of the parallel capacitor across the inrush reactor prevents sudden changes in voltages across the inrush reactor. Hence no step jump in voltage appears across the circuit breaker. There exists voltage and current oscillations in the parallel L - Cp tank circuit and this appears in the recovery voltage across the breaker. The recovery voltage hence constitutes a parallel transient frequency component (ω_P) and a series transient frequency component (ω_0). For the system parameters used in the study, the series transient frequency is approximately twice the parallel transient frequency. This results in a flat waveform close to the peak and a reduction in the peak of the transient recovery voltage seen across the breaker. This is hence an advantage.

In the three-phase circuits with the ungrounded three-phase capacitor bank configurations, the transient recovery voltage appearing across the first breaker pole to open is typically 1.5 times more than that appearing across the breaker in a single-phase circuit. There exists a neutral displacement voltage during unbalance conditions. The capacitor bank neutral is not at ground potential. This leads to the increased transient recovery voltage in the ungrounded three-phase circuits. Thus, breakers with sufficient rating to withstand the increased recovery voltage have to be installed.

Bibliography

[1] IEEE Std C37.011-2005, IEEE Application Guide for Transient Recovery Voltage for AC High-Voltage Circuit Breakers. Institute of Electrical and Electronics Engineers, New York, NY.

[2] R.C. Dugan, S. Santoso, and M.F. McGranaghan. *Electrical power systems quality*. McGraw-Hill Professional, 2003.

[3] A. Greenwood. *Electrical transients in power systems*. Wiley New York, 1991.

[4] RH Harner and RE Owen. Neutral Displacement of Ungrounded Capacitor Banks During Switching. *IEEE Transactions on Power Apparatus and Systems*, pages 1631–1638, 1971.

[5] M. Kandakatla, B.K. Rao, and G. Gajjar. Circuit Breaker Transient Recovery Voltage in Presence of Source Side Shunt Capacitor Bank. In *Joint International Conference on Power System Technology and IEEE Power India Conference, 2008. POWERCON 2008*, pages 1–6, 2008.

[6] D.L. Swindler, P. Schwartz, P.S. Hamer, and S.R. Lambert. Transient recovery voltage considerations in the application of medium-voltage circuit breakers. *IEEE Transactions on Industry Applications*, 33(2):383–388, 1997.

[7] L. van der Sluis and ALJ Janssen. Clearing faults near shunt capacitor banks. *IEEE Transactions on Power Delivery*, 5(3):1346–1354, 1990.

Vita

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